Radiative corrections to hard spectator scattering in $B \rightarrow \pi\pi$ decays

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ABSTRACT: We present the calculation of the next-to-leading corrections to the tree amplitudes which appear in the description of non-leptonic B-decays in the factorization approach. These corrections, together with radiative corrections to the jet functions, represent the full next-to-leading contributions to the dominant hard spectator scattering term generated by operators $O_{1,2}$ in the decay amplitudes. Using obtained analytical results we estimate $B \rightarrow \pi\pi$ branchings fractions in the physical (or BBNS) factorization scheme. We have also found that the imaginary part generated in the hard spectator scattering term is rather large compared to the imaginary part of the vertex contribution.

KEYWORDS: B-Physics, NLO Computations, Weak Decays.

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Contents

1. Introduction

The processes of non-leptonic decays of B mesons are considered as one of the most interesting topics at present time. They are sensitive to the physics of the standard model and provide a nice possibility to search for new physics effects. The channel of two π-mesons is very interesting because phenomenological analysis of the corresponding branchings and CP asymmetries can be done to a good accuracy in a model independent way [1]. Last few years $B \to \pi\pi$ branching fractions and CP-asymmetries have been measured by BABAR [2] and BELLE [3] collaborations.

Important progress has also been achieved in the theory. There was suggested a new approach which is based on the idea of QCD factorization. The factorization allows, in some sense, to constrain the strong interaction background in a model independent way and therefore provides a theoretical basis for analysis of $B-$decays which can be considered as an alternative to the traditional phenomenological fits.

The factorization theorem for nonleponic decays has been initially suggested in [4]. The statement has been proved by explicit calculations at the leading and next-to-leading orders. The general proof of the factorization to all orders can be done using the so-called soft-collinear effective theory (SCET) [5]. The application of SCET technique to the two mesons decays has been formulated in [6, 7]. The presence of two hard scales $\mu \sim m_b$ and $\mu \sim \sqrt{\Lambda m_b}$ leads to two steps matching QCD$\to$ SCET$_{\text{I}} \to$ SCET$_{\text{II}}$ with corresponding two independent coefficient functions which can be calculated systematically in perturbative QCD. The non-perturbative dynamics is described by the matrix elements

of the light-cone operators constructed from the fields of the SCET$_{II}$ effective theory. These unknown functions are universal for all processes and therefore can be constrained from the global analysis. Phenomenological applications to the nonleptonic decays based on the factorization have been considered in several papers. The QCD factorization approach (the so-called BBNS or physical scheme) was used in [9]. A different analysis on basis of SCET was suggested in [7, 8]. Although both approaches are based on the same theoretical idea they are different in the consideration of some phenomenological moments, see for instance discussion in [10].

An important question which appears in application of the factorization is applicability of perturbation theory at relatively moderate scale $\mu \sim \sqrt{\Lambda m_b} \sim 1-2\,\mathrm{GeV}$. This situation arises at the second step of the matching SCET$_I \to$ SCET$_{II}$. In order to answer this question, the next-to-leading calculations of the so called jet coefficient functions have been done in [12–14]. It was demonstrated [13] that the radiative corrections are large but, on the other side, do not indicate any problem for the applicability of the perturbative expansion. But the full next-to-leading contributions also include corrections to the hard coefficient functions which describe matching QCD to SCET$_I$ effective theory. A priory, such corrections could also be considered as a source of quite large contributions, especially for the color suppressed amplitudes in the BBNS analysis [13]. Therefore the second tail of the next-to-leading corrections, corresponding to the matching of QCD to SCET$_I$ at $\mu \sim m_b$ also have to be computed. An other important motivation for such a calculation is the observation that the imaginary part of hard spectator amplitude arises only from the radiative corrections. If it can produce sizable corrections to the CP-asymmetries then such contribution is very important for the phenomenological analysis.

Recently, such calculations have been carried out and results are presented in [15] for the graphical tree amplitudes and for the penguin amplitudes [16]. In this paper we present the calculations of the radiative corrections to the graphical tree amplitudes. Our results have been computed using different technical approach and can be considered as an independent derivation of the corresponding corrections.

Our paper has the following structure. In section I we introduce the basic notation and review, for convenience, the formulation of the factorization theorem for $B \to \pi\pi$ decays. In section II we discuss the matching from QCD to SCET$_I$. We define the basic set of SCET$_I$ operators and recalculate the leading order coefficient functions. The calculation of the one loop diagrams and results for the coefficient functions are given in section III. Section IV is devoted to the numerical estimates of the branching fractions. The discussion of some technical questions and the analytical results for the individual diagrams are presented in the appendix.

2. QCD factorization for $B \to \pi\pi$ decays

For the convenience we review shortly the basic QCD factorization approach suggested in [9]. The amplitudes of two pion decays are given by matrix elements

$$A_{\pi\pi} = \left\langle \pi(p')\pi(p) \left| H_{\mathrm{eff}} \right| \bar{B}(P) \right\rangle \tag{2.1}$$

with the effective Hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \lambda_u^{(d)} \left(C_1 O_1^u + C_2 O_2^u + \sum_{i=3}^{10} C_i O_i + C_{7\gamma} O_{7\gamma} + C_{8g} O_{8g} \right) + h.c. \qquad (2.2)$$

$$+ \frac{G_F}{\sqrt{2}} \lambda_c^{(d)} \left(C_1 O_1^c + C_2 O_2^c + \sum_{i=3}^{10} C_i O_i + C_{7\gamma} O_{7\gamma} + C_{8g} O_{8g} \right) + h.c. \qquad (2.3)$$

where $\lambda_u^{(d)} = V_{ub} V_{ud}^*$, $\lambda_c^{(d)} = V_{cb} V_{cd}^*$, C_i and O_i^p are coefficient functions and effective four-fermion operators respectively. In particular, the explicit expressions for the current-current operators are

$$O_1^u = (\overline{u}\, b)_{V-A} (\overline{d}u)_{V-A}, \quad O_2^u = (\overline{u}_\beta\, b_\alpha)_{V-A} (\overline{d}_\alpha u_\beta)_{V-A} \qquad (2.4)$$

$$O_1^c = (\overline{c}\, b)_{V-A} (\overline{d}c)_{V-A}, \quad O_2^c = (\overline{c}_\beta\, b_\alpha)_{V-A} (\overline{d}_\alpha c_\beta)_{V-A} \qquad (2.5)$$

where as usual $V - A = \gamma^\mu (1 - \gamma_5)$ and indices α, β stand for the color. The definitions of the remaining terms are standard and can be found, for instance, in [17]. Taking into account the structure of the effective Hamiltonian (2.3) the decay amplitudes $A_{\pi\pi}$ can be conveniently rewritten through the effective amplitudes α_i in the following way [9]:[1]

$$A_{\pi^+\pi^-} = -\lambda_u^{(d)} \frac{iG_F}{\sqrt{2}} M_B^2 f_\pi \left[\alpha_1 + \alpha_4^u + \alpha_{4,EW}^u \right] - \lambda_c^{(d)} \frac{iG_F}{\sqrt{2}} M_B^2 f_\pi \left[\alpha_4^c + \alpha_{4,EW}^c \right], \quad (2.6)$$

$$A_{\pi^0\pi^0} = \lambda_u^{(d)} \frac{iG_F}{\sqrt{2}} M_B^2 f_\pi \left[-\alpha_2 + \alpha_4^u - \frac{3}{2}\alpha_{3,EW}^u - \frac{1}{2}\alpha_{4,EW}^u \right]$$

$$+ \lambda_c^{(d)} \frac{iG_F}{\sqrt{2}} M_B^2 f_\pi \left[\alpha_4^c - \frac{3}{2}\alpha_{3,EW}^u - \frac{1}{2}\alpha_{4,EW}^c \right], \qquad (2.7)$$

where we have neglected the annihilation contributions. The amplitudes α_i describe the matrix elements of the different operators in (2.3). Namely, $\alpha_{1,2}$ gives the matrix elements of the current-current operators $O_{1,2}$, $\alpha_4^{u,c}$ and $\alpha_{3,4,EW}^{u,c}$ denote the QCD and Electro-Weak penguin contributions respectively. From the isospin symmetry one has

$$\sqrt{2} A_{\pi^0\pi^-} = A_{\pi^0\pi^0} + A_{\pi^+\pi^-} \qquad (2.8)$$

We used notation M_B for $B-$meson mass, f_π is pion decay constant and below $f_0 \equiv f_0(0) = f_+(0)$ denotes $B \to \pi$ transition form factors at $q^2 = 0$:

$$\left\langle \pi(p) \left| \overline{q}\gamma^\mu b \right| \bar{B}(P) \right\rangle = f_+(q^2) \left[P^\mu + p^\mu - \frac{M_B^2 - m_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{M_B^2 - m_\pi^2}{q^2} q^\mu . \qquad (2.9)$$

The amplitudes α_i^p include all dynamical information about the decays. In the limit of large $b-$quark mass $m_b \to \infty$ the QCD factorization approach makes it possible to calculate amplitudes α_i^p to the leading power accuracy. Let us consider the matrix elements $\alpha_{1,2}$ which provide dominant contribution to the branching fractions. Their expressions are given by

$$\alpha_i = f_0 \int_0^1 du\, V_i(u)\varphi_\pi(u) + \int_0^1 du\, \varphi_\pi(u) \int_0^1 dz\, T_i(u,z)\, \xi_\pi^{B1}(z), \qquad (2.10)$$

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[1]We have slightly changed the original notation removing f_0 from the normalization factor

$$\xi_\pi^{B1}(z) = f_B f_\pi \int_0^\infty d\omega \int_0^1 dx \ \phi_B(\omega) J(z,x,\omega) \ \varphi_\pi(x) \tag{2.11}$$

where functions V_i and T_i are the hard coefficient functions which can be computed in the perturbation theory order by order in QCD coupling α_S:[2]

$$V_i = V_i^{LO}(u) + \frac{\alpha_S}{2\pi} V_i^{NLO}(u,z,m_b/\mu_F) + \dots \tag{2.12}$$

$$T_i = T_i^{LO}(u) + \frac{\alpha_S}{2\pi} T_i^{NLO}(u,z,m_b/\mu_F) + \dots \tag{2.13}$$

The hard coefficient functions describe the hard subprocess in which quarks and gluons are highly virtual, with typical hard momenta $p_h^2 \sim m_b^2$. Performing integration over such fluctuations we reduce QCD to the effective theory SCET$_\mathrm{I}$ which however still contains large hard-collinear fluctuations of order $p_{hc}^2 \sim m_b \bar\Lambda$. Integrating over these degrees of freedom we reduce SCET$_\mathrm{I}$ to the low energy effective theory SCET$_\mathrm{II}$ which contains only collinear and soft particles with small $p_c^2 \sim p_s^2 \sim \bar\Lambda^2$ off-shell momenta. The coefficient function which appears at this step is the so-called jet-function $J(z,x,\omega)$:

$$J(z,x,\omega) = \alpha_S(\mu_{hc}) \ J^{LO}(z,x,\omega) + \dots \tag{2.14}$$

where the hard-collinear scale $\mu_{hc} \sim \sqrt{m_b \bar\Lambda}$. The soft physics is encoded by the matrix elements of SCET$_\mathrm{II}$ operators constructed from the soft and collinear fields. These matrix elements are parametrised by non-perturbative light-cone distribution amplitudes (LCDA) φ_π, ϕ_B and decay constants f_π, f_B. Their explicit definitions are given by

$$f_\pi \varphi_\pi(x) = i \int \frac{d\lambda}{\pi} e^{-i(2x-1)(p.n)\lambda} \left\langle \pi^-(p) \left| \bar{d}(\lambda n) \not{n} \gamma_5 u(-\lambda n) \right| 0 \right\rangle, \tag{2.15}$$

where $\bar{n}$ and n are the light cone vectors: $n^2 = \bar{n}^2 = 0, (n \cdot \bar{n}) = 2$ and pion decay constant defined as

$$\left\langle \pi^-(p) \left| \bar{d}(0) \not{n}\gamma_5 u(0) \right| 0 \right\rangle = -if_\pi(p.n), \tag{2.16}$$

that implies $\int dx \ \varphi_\pi(x) = 1$. $B-$meson LCDA is given by

$$F_{stat}(\mu) \sqrt{M_B} \phi_B(\omega) = -i \int \frac{d\lambda}{2\pi} e^{+i\omega\lambda} \left\langle 0 \left| \bar{q}(\lambda n) \not{n} \gamma_5 h_v(0) \right| \bar{B}(P) \right\rangle, \tag{2.17}$$

where $v = \frac{1}{2}(\bar{n}+n)$ is the velocity of $B-$meson at the rest frame. The M_B-independent decay constant $F_{stat}(\mu)$ [11] is defined as

$$F_{stat}(\mu) = \sqrt{M_B} \ f_B/K_F(\mu), \ K_F(\mu) = 1 + \frac{\alpha_S C_F}{4\pi} \left(3 \ln \frac{m_b}{\mu} - 2 \right) \tag{2.18}$$

where the physical decay constants f_B is given by

$$\left\langle 0 \left| \bar{q}\gamma^\mu \gamma_5 b \right| \bar{B}(P) \right\rangle = if_B M_B v^\mu. \tag{2.19}$$

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[2]In this paper we always assume that perturbative expansion of any quantity R is defined as $R = R^{LO} + \frac{\alpha_S}{2\pi} R^{NLO} + \dots$

As it was shown in [20, 21], the normalization integral for $\phi_B(\omega)$ and higher moments are not defined and therefore the non-local matrix element (2.17) can not be reduced to the local matrix element (2.18). This feature makes this function quite different from the standard LCDA 's of light mesons.

As we can see from equation (2.10) the jet function appears in the second term only. This term describes the hard spectator interaction. For the fist term in (2.10) the matching to SCET_{II} is not possible due to the overlap of the soft and collinear regions, see discussions [18, 19]. Such contribution is known as soft-overlap form factor. In the BBNS prescription this form factor is excluded using the so-called "physical scheme". In this approach the soft-overlap form factor is rewritten as a sum of physical form factor f_0 and of hard spectator scattering contribution (the details are given below in the text).

Explicit expressions for the hard coefficient functions read [9] $(i \pm 1 \equiv i + (-1)^{i+1})$

$$V_i(u) = \left(C_i + \frac{C_{i\pm 1}}{N_c} \right) + \frac{\alpha_S}{2\pi} V_i^{NLO}(u) + O(\alpha_S^2), \tag{2.20}$$

$$V_i^{NLO}(u) = \frac{C_{i\pm 1}}{N_c} V(u), \tag{2.21}$$

$$V(u) = \frac{1}{2} C_F \left(12 \ln \frac{m_b}{\mu_h} - 18 + 3 \left(\frac{1-2u}{1-u} \ln u - i\pi \right) + \tag{2.22}$$

$$\left[2\text{Li}_2[u] - \ln^2 u + \frac{2 \ln u}{1-u} - (3 + 2i\pi) \ln u - (u \leftrightarrow \overline{u}) \right] \right).$$

$$T_i(u, z) = -\frac{C_{i\pm 1}}{N_c} \frac{1}{1-u} + \frac{\alpha_S}{2\pi} T_i^{NLO}(u, z) + O(\alpha_S^2), \tag{2.23}$$

For the jet function in our notation we have

$$J(z, x, \omega) = \alpha_S \left\{ \left[-\pi \frac{C_F}{N_c} \frac{\delta(x-z)}{\overline{x} \, \omega m_b} \right] + \frac{\alpha_S}{2\pi} J^{NLO}(z, x, \omega) + \dots \right\}, \tag{2.24}$$

The next-to-leading order expression for $J^{NLO}(z, x, \omega)$ has been recently obtained in several papers $[12-14]$ and we shall not present it here.

3. Calculation of the hard coefficient functions

The aim of this section is to discuss some details relevant for our calculation. We shall reproduce the leading order results for hard coefficient functions T_i quoted in eq. (2.23). As it was discussed in the previous section, T_i is associated with matching QCD to the effective theory SCET_{I}. Technical details of such calculations have already been discussed in $[23-25]$ for the case of heavy-to-light currents.

First, let us fix the basis of SCET_{I} operators relevant for our case. We introduce two operators with approprite flavor q and chiral structure:

$$J^{(A0)}(s) = (\bar{q}_n W_c)_s \left(1 - \frac{i \overleftarrow{\slashed{\partial}}}{i \, \bar{n} \overleftarrow{\partial}} \right) h_v, \tag{3.1}$$

$$J^{(B1)}(s) = \frac{1}{m_b} (\bar{q}_n W_c)_s (W_c^\dagger i \slashed{D}_{\perp c} W_c)_{(-s)} (1 - \gamma_5) h_v. \tag{3.2}$$

where we have accepted the notation introduced in [26]. The light quarks are supposed to be collinear fields in SCET approach $\not{n} q_n(x) = 0$, h_v is HQET field. Notation W_c is used for the hard-collinear Wilson line involving only $\bar{n} \cdot A_c$ component of the collinear gluon field:

$$(\bar{q}_n W_c)_s = \bar{q}_n(s\,\bar{n}) P \exp\left\{ ig \int_{-\infty}^{0} du\, \bar{n} A_c[(s+u)\,\bar{n}] \right\}. \tag{3.3}$$

Matrix elements of these operators between physical particles define two SCET$_I$ form factors:

$$\left\langle \pi^+(p) \left| (\bar{d}_n W_c)\, h_v \right| \bar{B}(P) \right\rangle = m_b\, \xi_\pi, \tag{3.4}$$

$$\left\langle \pi^+(p) \left| J^{(B1)}(s) \right| \bar{B}(P) \right\rangle = m_b \int_0^1 dz\, e^{is(2z-1)m_b}\, \xi_\pi^{B1}(z), \tag{3.5}$$

where dependence of the form factors on mass m_b is implied.

In order to obtain the factorization formula (2.10) one has to perform matching of the effective Hamiltonian (2.3) to the operators in SCET$_I$ (3.1) and (3.2) . We shall focus our attention on the contributions of the current-current operators $O_{1,2}$ because they provide dominant part of the two body decay amplitude. Then for the matrix element $A_{\pi\pi}$ (2.1) we obtain

$$A_{\pi\pi}^i = \frac{G_F}{\sqrt{2}} \lambda_u^{(d)} \left\langle (\pi\pi)_i \left| C_1 O_1^u + C_2 O_2^u \right| \bar{B} \right\rangle = -\frac{iG_F}{\sqrt{2}} M_B^2 f_\pi(\,\alpha_i\,), \tag{3.6}$$

$$\alpha_i = \left(\, \xi_\pi\, v_i * \varphi_\pi + \varphi_\pi * t_i * \xi_\pi^{B1} \right), \tag{3.7}$$

where v_i and t_i denote hard coefficient functions and by asterisk $*$ we denote, for simplicity, convolution integrals. The index i is introduced to distinguish two possible final states $(\pi\pi)_{i=1} = \pi^+\pi^-$, $(\pi\pi)_{i=2} = \pi^0\pi^0$. Corresponding matrix elements define amplitudes α_1 and α_2 respectively.

In the physical scheme one has to express SCET$_I$ form factor ξ_π through the physical form factor $f_+(0) = f_0$ [13]:

$$\xi_\pi = \frac{1}{C^{A0}} f_0 - \frac{1}{C^{A0}} C^{B1} * \xi_\pi^{B1}, \tag{3.8}$$

where C^{A0} and C_+^{B1} are the hard coefficient functions which appear in matching of the scalar heavy-light QCD current to the operators (3.1), (3.2):

$$\bar{q}\, b = C^{A0} * J^{A0} + C^{B1} * J^{B1} + O(1/m_b) \tag{3.9}$$

Inserting equation (3.8) into (3.6) we obtain

$$\alpha_i = f_0\, v_i/C^{A0} * \varphi_\pi + \varphi_\pi * \left(t_i - v_i\, C^{B1}/C^{A0} \right) * \xi_\pi^{B1}, \tag{3.10}$$

Comparing this expression with equation (2.10) we find

$$V_i(u) = v_i(u)/C^{A0}, \tag{3.11}$$

$$T_i(u,z) = t_i(u,z) - v_i\,(u)\, C^{B1}(z)/C^{A0}\,. \tag{3.12}$$

These expressions define precisely the coefficient functions V_i and T_i in the physical scheme through the matching coefficient v_i and t_i of the effective operators $O_{1,2}$. Introducing perturbative expansions for the coefficient functions:

$$C^{A0} = 1 + \frac{\alpha_S}{2\pi} C^{A0}_{NLO} + \dots, \tag{3.13}$$

$$C^{B1}_+ = C^{B1}_{LO} + \frac{\alpha_S}{2\pi} C^{B1}_{NLO} + \dots, \tag{3.14}$$

$$v_i(u) = v^{LO}_i(u) + \frac{\alpha_S}{2\pi} v^{NLO}_i(u) + \dots, \tag{3.15}$$

$$t_i(u, z) = t^{LO}_i(u, z) + \frac{\alpha_S}{2\pi} t^{NLO}_i(u, z) + \dots. \tag{3.16}$$

we obtained for the functions V_i and T_i in (2.12) and (2.13)

$$V^{LO}_i = v^{LO}_i, \tag{3.17}$$

$$V^{NLO}_i = v^{NLO}_i - v^{LO}_i C^{A0}_{NLO}, \tag{3.18}$$

$$T^{LO}_i = \left(t^{LO}_i - v^{LO}_i C^{B1}_{LO} \right), \tag{3.19}$$

$$T^{NLO}_i = \left(t^{NLO}_i - v^{LO}_i C^{B1}_{NLO} \right) - C^{B1}_{LO} V^{NLO}_i. \tag{3.20}$$

One can observe that subtraction terms $-v^{LO}_i C^{A0}_{NLO}$ in equation (3.18), $-v^{LO}_i C^{B1}_{LO}$ and $-v^{LO}_i C^{B1}_{NLO}$ in (3.19) and (3.20) can cancel the contributions of the "factorizable" diagrams which can be considered as appropriate product of the two matrix elements:

$$\left\langle (\pi\pi)_i \left| O^u_{1,2} \right| \bar{B} \right\rangle_{\text{fact}} \sim \left\langle \pi \left| \bar{q}\Gamma q \right| 0 \right\rangle \left\langle \pi \left| \bar{q}\Gamma b \right| \bar{B} \right\rangle \tag{3.21}$$

If it is fulfilled, then such diagrams can be ignored in the calculations of the coefficient functions T^{LO}_i and T^{NLO}_i.

In our paper we shall obtain the coefficient functions T_i computing matrix elements with quarks and gluons. For that purpose we define the perturbative analogs of the discussed form factor ξ^{B1}_π and LCDA φ_π. To define SCET$_I$ form factor let us consider as external state a hard-collinear quark and a gluon with momenta p_1 and p_2 respectively:

$$p_1 = z\, p + (p_1 n)\frac{\bar{n}}{2} + p_{1\perp}, \tag{3.22}$$

$$p_2 = \bar{z}\, p + (p_2 n)\frac{\bar{n}}{2} + p_{2\perp}, \tag{3.23}$$

where $p = m_b \frac{n}{2}$, $\bar{z} = 1 - z$ and the other components are $(n \cdot p_{1,2}) \sim \bar{\Lambda}$, $|p_{1,2\perp}| \sim \sqrt{\bar{\Lambda} m_b}$ as it necessary for the hard-collinear momenta. Then we define
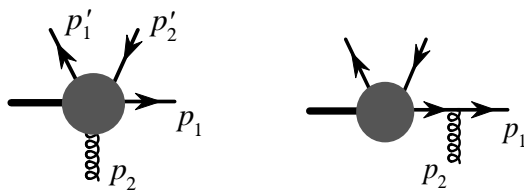
$$\left\langle q(p_1)g(p_2) \left| J^{(B1)}(s) \right| h_v \right\rangle = m_b \int_0^1 dz\, e^{is(2z-1)(p.\bar{n})}\xi_B\, \theta(z). \tag{3.24}$$

The factor ξ_B denotes the relevant combination of the quark spinors and gluon polarization vector: $\xi_B = \frac{g}{m_b^2}\bar{q}\slashed{\epsilon}_g(1 - \gamma_5)h_v$.[3] We assume that the final gluon is transversely polarized

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[3]Symbols $\bar{q}$ and h_v denote the quark spinors in this formula.

Figure 1: Graphical representation of the QCD matrix elements which we use to compute T_i. The right graph corresponds to the QCD diagrams with emission of collinear gluon by collinear outgoing quark. Such graphs include contribution from operator $J^{(A0)}$ with quark-gluon vertex from the leading order SCET lagrangian.

with the polarization vector e_g, the color indices are not shown for simplicity. Performing simple tree level calculation one finds

$$\theta^{LO}(\tau) = \delta(z - \tau). \tag{3.25}$$

In order to define the perturbative analog of the pion LCDA we consider quark-antiquark state with collinear momenta

$$p_1' = up' + p_{1\perp}', \quad p_2' = \bar{u}p' + p_{2\perp}', \quad p' = m_b \frac{\bar{n}}{2}. \tag{3.26}$$

Then[4]

$$\langle p_1' p_2' | \bar{u}(\lambda n) \, \not{n} \, \gamma_5 \, d(-\lambda n) | 0 \rangle = -i \, f_P(p'n) \int_0^1 dx e^{i\lambda(2x-1)(p'n)} \varphi_P(x), \tag{3.27}$$

where we introduced notation $f_P = i \, \bar{u} \not{n} \gamma_5 u/m_b$. Again, from the leading order calculation one obtains

$$\varphi_P^{LO}(u) = \delta(u - x). \tag{3.28}$$

In order to calculate coefficient functions T_i we introduce the matrix elements describing the decay of the b-quark into three quarks and gluon

$$\langle p_1' \, p_2', \, p_1 \, p_2 \, | C_1 O_1 + C_2 O_2 | \, b_v \rangle_{\text{nf}} - \left\langle p_1', p_2' \, p_1 p_2 \left| T \left\{ t_1 * J^{(A0)}, \mathcal{L}_{hc}^{int} \right\} \right| b_v \right\rangle_{\text{nf}} = i m_b^2 \, f_P \xi_B \varkappa_i^T \tag{3.29}$$

and parameterized by form factors $\varkappa_i^T$ respectively.

By the subscript "nf" we indicate that we exclude the factorizable diagrams (3.21) which, as expected, cancel in the transition to the physical scheme. The subtraction term in the left side of eq. (3.29) represents the admixture of the operator $J^{(A0)}$ with one insertion of the interaction vertex $\mathcal{L}_{hc}^{int}$ from the LO hard-collinear SCET lagrangian [24, 25]. Such contribution describes the emission of collinear gluon from collinear quark and are present only in the diagrams with topology, given in figure 1. Practically this subtraction can be easily done by the substitution

$$\bar{u}(p_1)\not{e}_g \frac{i\not{p}}{p^2} \longrightarrow \bar{u}_n \not{e}_g \frac{\bar{\not{n}}}{2} \frac{i}{(p\bar{n})} \tag{3.30}$$

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[4]For simplicity, we do not introduce here the collinear SCET fields following standard QCD notation.
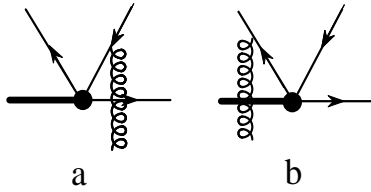
Figure 2: Leading order diagrams for the coefficient functions T_i .

where $\bar{u}(p_1)$ is the wave function of collinear quark in full QCD and $\bar{u}_n$ denotes the hard-collinear spinor in SCET.

From the factorization we expect that

$$\varkappa_i^T = \int_0^1 du \ \varphi_P(u) \int_0^1 dz \ \widetilde{t}_i(u,z) \ \theta(z), \tag{3.31}$$

where by tilde we denote coefficient functions of nonfactorizable diagrams.

Consider, as example, calculation of T_i^{LO}. Relevant tree diagrams at the leading order are shown in figure 2. Two diagrams with emission of a gluon from the bottom lines represent the factorisable contributions (3.21) which cancel against $-v_i^{LO} C_{LO}^{B1}$ in (3.19).

Straightforward calculation gives (after Fierz transformation)

$$D_a = 0, \tag{3.32}$$

$$D_b = i \ m_b^2 \ f_P \ \xi_B \left(-\frac{C_{i \pm 1}}{N_c} \frac{1}{\bar{u}} \right). \tag{3.33}$$

Comparing with eq. (3.31) we obtain:

$$\int_0^1 du \ \varphi_P^{LO}(u) \int_0^1 dz \ T_2^{LO}(u,z) \ \theta^{LO}(z) = \left(i \ m_b^2 \ f_P \xi_B \right)^{-1} D_b \tag{3.34}$$

Inserting the leading order expressions for the form factor θ^{LO} and LCDA φ_P^{LO} we find the leading order hard coefficient functions:

$$T_i^{LO}(u,z) = -\frac{C_{i \pm 1}}{N_c} \frac{1}{1-u} \tag{3.35}$$

As one can observe, LO results have no $z-$ dependence. In the next section we use the same technique to compute the next-to-leading order corrections.

4. Calculation of the coefficient functions T_i in the next-to-leading order

Corresponding one-loop diagrams are shown in figure 3. Factorisable diagrams, in the sense of (3.21), are not shown for simplicity. These are the diagrams where the external gluon is emitted from one of the bottom quark lines and the virtual gluon connects only the bottom (upper) quark lines but not upper and bottom. For the case of form factor α_1, these diagrams naturally reproduce subtraction term $v_1^{LO} C_{NLO}^{B1}$ in (4.28) and therefore cancel.
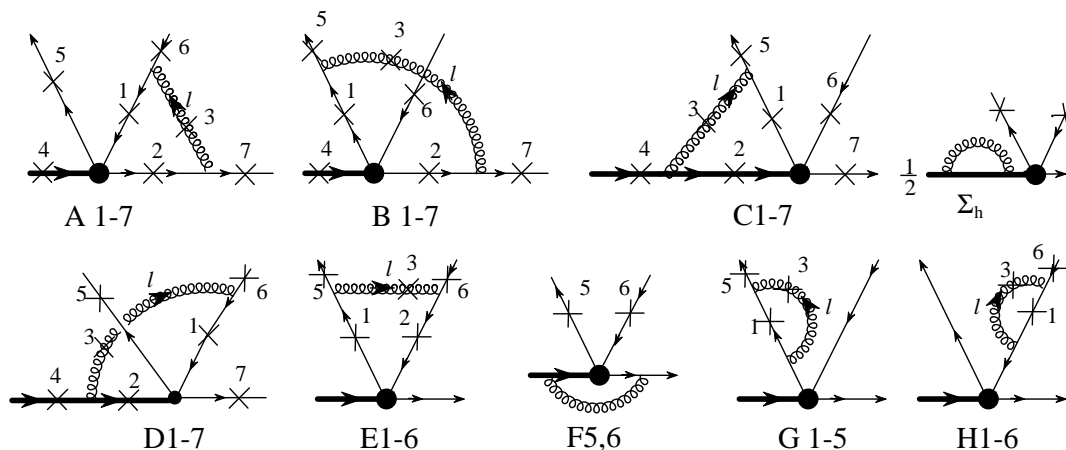
Figure 3: One loop diagrams which have to be computed in QCD. The crossed line denotes the emission of the outgoing gluon and the number gives the index of the corresponding diagram. The momentum flow is shown only for one line with the loop momentum l . We skip for simplicity the diagrams with the light quark wave function renormalization.

But analogous situation for the α_2 is more involved because of the different Dirac structure of the operator vertex. The problem is that corresponding $UV-$ divergent diagrams in dimensional regularization can not be represented exactly in the factorised form (3.21) because Fierz identities can not be used to regularized diagrams. Therefore one has to check the exact cancellation against $v_2^{LO} C_{NLO}^{B1}$ after $UV-$renormalization. We have fond that in accordance with subtraction scheme, described below in the text, such cancellation is exact. Therefore we shall not discuss these diagrams further.

From the factorization we expect that form factors $\varkappa_i^T$ describing the matrix elements (3.29) of the renormalized QCD operators can be represented as a sum of three contributions:

$$
\begin{aligned}
\left(\varkappa_i^T\right)_{NLO} &= \varphi_P^{LO}(x') * \widetilde{t}_i^{NLO}(x',z') * \theta^{LO}(z') \\
&\quad + \varphi_P^{NLO}(x') * \widetilde{t}_i^{LO}(x',z') * \theta^{LO}(z') + \varphi_P^{LO}(x') * \widetilde{t}_i^{LO}(x',z') * \theta^{NLO}(z') \quad (4.1)
\end{aligned}
$$

where φ_P^{NLO} and θ^{NLO} denote the contributions of the renormalized matrix elements (3.24) and (3.27) in the next-to-leading order. The three contributions in (4.1) can be associated with four integration regions in the loop integrals. The *hard* region $k_i \sim m_b$, $k^2 \sim m_b^2$ provides contributions to the $\widetilde{t}_i^{NLO}(x',z')$, the *collinear to p'* must be associated with contributions to $\varphi_P^{NLO}(x')$, the *soft* $k_i \sim \bar{\Lambda}$, $k^2 \sim \bar{\Lambda}^2$ and *collinear to p* regions can be associated with the $\theta^{NLO}(z')$. Substituting in (4.1) the explicit expressions for the φ_P^{LO}, $\theta^{LO}(z')$ and $\widetilde{t}_i^{LO}(x',z')$ we obtain

$$
\widetilde{t}_i^{NLO}(u,z) = \left(\varkappa_i^T\right)_{NLO} + \frac{C_{i\pm1}}{N_c} \int_0^1 dx' \frac{\varphi_P^{NLO}(x')}{1-x'} + \frac{C_{i\pm1}}{N_c} \frac{1}{\bar{u}} \int_0^1 dz' \theta^{NLO}(z') . \quad (4.2)
$$

Inserting this expression into eq. (3.20) and substituting $C_{LO}^{B1} = -1$ [22, 26] we find for the

NLO coefficient functions in physical scheme:

$$T_i^{NLO}(u,z) = \left(\varkappa_i^T\right)_{NLO} + \frac{C_{i\pm1}}{N_c}\int_0^1 dx' \frac{\varphi_P^{NLO}(x')}{1-x'} + \frac{C_{i\pm1}}{N_c}\frac{1}{\bar{u}}\int_0^1 dz'\theta^{NLO}(z') + V_i^{NLO}. \quad (4.3)$$

Let us now discuss the calculation of different terms appearing in (4.3). To perform the calculations of the diagrams in figure 3 one has to introduce regularization for the ultraviolet (UV) and infrared (IR) divergencies. We shall use dimensional regularization with $D = 4-2\varepsilon$ to subtract $UV-$divergencies. To compute the $UV-$divergent subdiagrams of the four-fermion operators we use $NDR-$scheme with the anticommuting γ_5 matrix. Note that Fierz identities then can be used only for the renormalized matrix elements in four dimensions. For the $IR-$divergencies we use regularization by off-shell external momenta. Such regularization makes possible to perform all manipulation with Dirac algebra for the $UV-$finite integrals in four dimensions.

As an illustration, let us consider a contribution of some diagram D_X which can be represented in the following way

$$D_X = \int d^D l \ \ \bar{u}\Gamma_1 b_v \ \bar{d}\Gamma_2 u, \quad (4.4)$$

where $u,\bar{u},b_v,\bar{d}$ are quark spinors of given flavor and matrices $\Gamma_1\otimes\Gamma_2$ denote some momentum dependent expressions. Contraction of spinor indices can be organized in two different ways which correspond to the amplitudes α_1 or α_2. As an example below we consider the calculation of α_2. The same technique also was used for α_1.

All graphs can be divided into two groups: $UV-$divergent and $UV-$finite. $UV-$divergent subgraphs, which appear in graphs $G\{1,3,5\},H\{1,3,6\}$ represent usual divergencies of the QCD Green functions. They are removed by QCD Lagrangian counterterms. $UV-$subgraphs in diagrams $A\{4,5,6\}$, $B\{4,5,6\}$, $C\{4,5,6\}$, $D\{4,5,6\}$ and $F\{5,6\}$ describe renormalization of the four-fermion operators $O_{1,2}$. Calculation of the corresponding $UV-$divergent integrals must to be performed in $D = 4-2\varepsilon$. A typical expression for the integrand of $UV-$divergent graph can be written as

$$\bar{u}\Gamma_1 b_v \ \bar{d}\Gamma_2 u \ = N_2^{\mu\nu}\frac{l_\mu l_\nu}{D[l]} + N_1^\mu\frac{l_\mu}{D[l]} + N_0\frac{1}{D[l]} \quad (4.5)$$

where the l-independent functions $N_i^{\mu\cdots}$ contain Dirac structures and spinors from the numerator, $D[l]$ denotes the denominator, which behaves at large Euclidian momentum l as $D[l] \sim (l^2)^3$. Such situation is usual for the $UV-$divergent graphs mentioned above, except only diagrams with quark self-energy subgraphs. Substituting (4.5) in (4.4) we obtain

$$D_X = N_2^{\mu\nu}J[l_\mu l_\nu] + N_1^\mu J[_\mu] + N_0 J_0, \quad (4.6)$$

where we introduced

$$J[l_\mu l_\nu] = \int d^D l \frac{l_\mu l_\nu}{D[l]} \quad (4.7)$$

and similar for others integrals. Taking into account the behavior of the denominator at large momentum $D[l] \sim (l^2)^3$, it's clear that only $J[l_\mu l_\nu]$ is $UV-$divergent. The other three

integrals can have only IR-divergencies, regulated by the off-shell momenta and therefore can be considered in $D = 4$. One can easily express the tensor integral $J[l_\mu l_\nu]$ through scalar integrals:

$$J[l_\mu l_\nu] = g_{\mu\nu} J_1 + (n_\mu \bar{n}_\nu + \bar{n}_\mu n_\nu) J_2 \tag{4.8}$$

with

$$J_1 = \frac{1}{2(1 - \varepsilon)} \left(J[l^2] - J[(l \cdot n)(l \cdot \bar{n})] \right), \tag{4.9}$$

$$J_2 = \frac{1}{4(1 - \varepsilon)} \left((2 - \varepsilon) J[(l \cdot n)(l \cdot \bar{n})] - J[l^2] \right). \tag{4.10}$$

Both scalar integrals $J[l^2]$ and $J[(l \cdot n)(l \cdot \bar{n})]$ have UV−poles. But in the coefficient J_2 the poles cancel. Hence we must contract $g_{\mu\nu}$ with Dirac structure $N_2^{\mu\nu}$ in $D = 4 - 2\varepsilon$ and expand the obtained expression up to terms $\sim \varepsilon$. The reduction of all one-loop Dirac structures to tree spinor combinations can be performed with the help of NDR prescription

$$\gamma^\mu \gamma^{\mu_1} \gamma^{\mu_2} (1 - \gamma_5) \otimes \gamma_\mu \gamma_{\mu_1} \gamma_{\mu_2} (1 - \gamma_5) = 4(4 - \varepsilon) \gamma^\mu (1 - \gamma_5) \otimes \gamma_\mu (1 - \gamma_5) \tag{4.11}$$

Let us here make following observation. Computing the integrals $J[l^2]$ and $J[(l \cdot n)(l \cdot \bar{n})]$ we do not remove the IR−regularization because these integrals can also happen to be IR−divergent and we must avoid the mixing of UV− and IR−poles. However, we observed, that one can always choose some "convenient" momentum flow for which these integrals have only UV−divergences and free from IR−singularities. In this case one can drop the IR-regularization making the calculations more simple.

Defining UV−pole of the coefficient J_1 as

$$\text{pole part} [J_1] = \frac{1}{\varepsilon} Z^{UV} \tag{4.12}$$

we write UV−divergent contribution for the D_X as

$$D_X^{UV} = \frac{1}{\varepsilon} Z^{UV} \lim_{\varepsilon \to 0} N_2^{\mu\nu} g_{\mu\nu} = i m_b^2 f_P \xi_B \frac{\alpha_S}{2\pi} X_{col} JX_{UV} \tag{4.13}$$

where X_{col} is the color factor of the diagram and JX_{UV} represents some UV−divergent expression. In appendix we provide explicit expressions for X_{col} and JX_{UV} for each diagram.

The singular contributions D_X^{UV} are removed by tree diagrams with operators renormalization constants and by QCD counterterms, see details in the appendix. After that, performing Fierz transformation, we obtaine trivial contributions from all the diagrams with topology $A5 - G5$ and $G\{1, 3\}$.

The similar calculation of the self-energy one-loop diagrams is much simple because they have only UV−poles and can be easily reduced to the tree Dirac structures. Let us mention, that we must consider also the diagrams with the one loop corrections to the wave functions which are not listed in the figure 3, except for the heavy quark Σ_b. We discuss these contributions in the appendix.

Finally, the finite expression for the diagram under consideration can be written as

$$[D_X]_R = [N_2^{\mu\nu} g_{\mu\nu} J_1]_R + N_2^{\mu\nu}(n_\mu \bar{n}_\nu + \bar{n}_\mu n_\nu) J_2 + N_1^\mu J[l_\mu] + N_0 J_0, \qquad (4.14)$$

where $[\ldots]_R$ denotes renormalized quantity. Remaining integrals have only IR−divergencies: collinear and soft. Calculation of such contributions is the same for all diagrams, with and without UV−subgraphs. Because we use off-shell regularization, we put $D = 4$ and perform projections on the pseudoscalar state with momentum p':

$$u_\beta^a \, \bar{u}_\alpha^b = (-i) \, f_P \frac{\delta^{ab}}{N_c} \, [\not{p}' \gamma_5]_{\alpha\beta}, \qquad (4.15)$$

and on the SCET$_\text{I}$ operator J^{B1}:

$$\bar{d}_\alpha^a b_{v\beta}^c = \bar{d}_n t^A \gamma_\perp^\sigma h_v \, [V_{B1}^\sigma]_{\beta\alpha} \otimes t_{ca}^A + \, \ldots, \qquad (4.16)$$

$$[V_{B1}^\sigma]_{\beta\alpha} = \frac{1}{2} \left[\not{n} \, \gamma_\perp^\sigma - \frac{1}{2} \not{\bar{n}} \gamma_\perp^\sigma \not{n} \right]_{\beta\alpha} \qquad (4.17)$$

where t^A are the standard color matrices satisfying $\text{Tr}(t^A t^B) = \frac{1}{2}\delta_{AB}$ and dots stand for the irrelevant spin and color structures. It is convenient also to insert parametrisation for the vector integral

$$J[l_\mu] = \frac{1}{2} n_\mu J[(l \cdot \bar{n})] + \frac{1}{2} \bar{n}_\mu J[(l \cdot n)]. \qquad (4.18)$$

After calculation of traces and contractions we arrive to the expression for $[D_X]_R$ which can be written as a sum scalar integrals

$$[D_X]_R = i m_b^2 \, f_P \xi_B \frac{\alpha_S}{2\pi} \, [J_X]_R, \qquad (4.19)$$

$$[J_X]_R = \left[a_{21}(\varepsilon) J[l^2] + a_{22}(\varepsilon) J[(l \cdot n)(l \cdot \bar{n})] \right]_R + b_{11} J[(l \cdot \bar{n})] + b_{12} J[(l \cdot n)] + c_0 J_0. \qquad (4.20)$$

with some coefficients a_{2i}, b_{1i} and c_0. The coefficients a_{2i}, in front of UV−divergent integrals are computed in dimension D and therefore depend on ε.

The sum of the integrals $[J_X]_R$ gives the formula for the form factor $\varkappa_i^T$:

$$\left(\varkappa_i^T \right)_{NLO} = \sum_X \, [J_X]_R. \qquad (4.21)$$

Let us stress again, that expression with brackets $[\ldots]_R$ in (4.20) denotes the renormalized quantity. We simply have rewritten the coefficients $J_{1,2}$ in terms of the corresponding integrals (4.10), (4.9) and introduced ε-dependent factors a_{2i} which arise from the calculations in DR. Assume for simplicity that UV−divergent integrals are free from IR−singularities. As we have discussed above, such situation can be realized for each diagram. Then contributions associated with these integrals are simply some finite expressions which do not depend on IR−regularization parameters. Our task now is to compute the remaining integrals $J[(l \cdot n)]$, $J[(l \cdot \bar{n})]$ and J_0.

Evaluation of these four-dimensional integrals can be performed with the technique known as expansion by regions, see for instance [27]. The dominant regions have been discussed above in the text. Hence, instead of one finite integral we obtain the sum of more simple but divergent integrals. According to general prescription, we use dimensional regularization in order to regularize the simplified integrals in each region. Therefore, in accordance with the dominant regions we have find following general decomposition of $[J_X]_R$

$$[J_X]_R = (J_X)_{hard} + (J_X)_{coll-p} + (J_X)_{coll-p'} + (J_X)_{soft} \ . \tag{4.22}$$

Taking into account that $IR-$divergencies are related with collinear and soft regions we find:

$$(J_X)_{hard} = \left[a_{21}(\varepsilon)J[l^2] + a_{22}(\varepsilon)J[(l \cdot n)(l \cdot \bar{n})] \right]_R + $$
$$(b_{11}J[(l \cdot \bar{n})] + b_{12}J[(l \cdot n)] + c_0 J_0)_{hard} \ , \tag{4.23}$$

$$(J_X)_{coll, soft} = (b_{11}J[(l \cdot \bar{n})] + b_{12}J[(l \cdot n)] + c_0 J_0)_{coll, soft} \ . \tag{4.24}$$

The hard region contributions in DR have $IR-$poles instead of $IR-$logarithms as in the case of off-shell regularization. The collinear and soft contributions have only $UV-$divergencies (off-shell regularization works in $IR-$regions). But the sum of all terms must be finite because of cancellation between $UV-$ and $IR-$poles.

The contributions from the collinear and soft regions depend on the external off-shell momenta which we use as $IR-$regulators in the original integral J_X. Inserting decomposition (4.22) into (4.21) and then into (4.3) we must recover the cancellation of the soft $IR-$scales. This is a good check of the factorization in the next-to-leading order. It is convenient to define the quantity:

$$S = \sum_X \left\{ (J_X)_{coll-p} + (J_X)_{coll-p'} + (J_X)_{soft} \right\} $$
$$+ \frac{C_{i\pm1}}{N_c} \left(\int_0^1 dx' \frac{\varphi_P^{NLO}(x')}{1-x'} + \frac{1}{\bar{u}} \int_0^1 dz' \theta^{NLO}(z') \right) \ . \tag{4.25}$$

Then taking into account that φ_P^{NLO} and θ^{NLO} are defined as matrix elements of the renormalized operators (the $UV-$poles are subtracted)[5] we expect that the answer for S can be represented as a sum of $UV-$poles and scale independent constant:

$$S = \frac{1}{\varepsilon^2}Z_2 + \frac{1}{\varepsilon}Z_1 + Z_0. \tag{4.26}$$

The poles, arising from the collinear and soft integrals in (4.26), must cancel against $IR-$poles appearing in the hard integrals $(J_X)_{hard}$ in (4.22). It is clear that the residues $Z_{1,2}$ can be related to the corresponding renormalization constant of the LCDA φ_P and form factor θ.

In order to obtain the finite terms Z_0 one has to perform calculation of the collinear and soft integrals, matrix elements (3.24) and (3.27) and compute the sum. It is clear

---

[5]Note that one must apply the same $IR-$regularization for the matrix elements which define φ_P^{NLO} and θ^{NLO}.

that both calculations overlap and this may be used to simplify derivation of the term Z_0. The SCET$_I$ non-renormilized matrix elements must be computed in DR, as usually, with D-dimensional Dirac algebra. But the structure of numerators of the diagrams for the matrix elements are relatively simple and their reduction to the basic combinations can be evaluated without any special prescriptions. The calculation of the form factor (3.24) is more complicate in comparison with pure collinear pion LCDA (3.27) because certain diagrams generate $UV-$poles of second order. But corresponding diagrams always have very simple numerators. Performing reduction of Dirac algebra to the basic structures one can again rewrite expressions for the matrix elements in terms of the scalar integrals which are similar to those appearing from the collinear and soft regions in QCD diagrams. Now the coefficients in front of these integrals are ε-dependent. We have found following representation for the bare NLO SCET$_I$ matrix elements:

$$\frac{C_{i\pm1}}{N_c} \left(\int_0^1 dx' \frac{\varphi_P^{NLO}(x')}{1-x'} + \frac{1}{\bar{u}} \int_0^1 dz' \theta^{NLO}(z') \right)_{bare} =$$
$$-\sum_X \left\{ (J_X)_{coll-p} + (J_X)_{coll-p'} + (J_X)_{soft} \right\} + \varepsilon \sum_X I_X \qquad (4.27)$$

where I_X are some $UV-$divergent integrals. These integrals have only first order poles in ε. All integrals with second order poles are absorbed into the first sum in rhs (4.27). Therefore taking into account that integrals I_X have coefficient ε we obtain finite contribution from the second term in rhs (4.27). The factor ε in the numerators appears, as a rule, from the reduction of D-dimensional structures in the diagrams to the basic factors ξ_B and f_P. Combining (4.27) with (4.25) and taking into account $UV-$counteterterms for the SCET$_I$ operators we obtain representation (4.26). It is clear that terms εI_X provide contributions to the constant Z_0 in (4.25).

Substitution (2.21) and (4.26) in formula for the coefficient function (4.3) gives:

$$T_i^{NLO}(u,z) = \sum_X (J_X)_{hard} + \frac{1}{\varepsilon^2} Z_2 + \frac{1}{\varepsilon} Z_1 + Z_0 + \frac{C_{i\pm1}}{N_c} V(u). \qquad (4.28)$$

This is our final working formula. It is convenient to rewrite the pole contributions as

$$Z_1 = (Z_1)_{s/coll-p} + (Z_1)_{coll-p'} \qquad (4.29)$$

where $(Z_1)_{s/coll-p}$ and $(Z_1)_{coll-p'}$ is associated with the form factor θ^{NLO} (soft and collinear $-p$ regions) and LCDA φ_P^{NLO} (collinear$-p'$ region) respectively. Calculation of the relevant integrals (details are considered in the appendix) gives

$$(Z_1)_{coll-p'} = C_F \left(-\frac{C_{i\pm1}}{N_c} \right) \frac{1}{\bar{u}} \left(2 + \ln \bar{u} \right), \qquad (4.30)$$

$$\frac{1}{\varepsilon^2} Z_2 + \frac{1}{\varepsilon} (Z_1)_{s/coll-p} = \left(-\frac{C_{i\pm1}}{N_c} \frac{1}{\bar{u}} \right) \left(-\frac{1}{\varepsilon^2} \frac{C_F}{2} (1 + 2\varepsilon \ln [\mu/(p\bar{n})]) \qquad (4.31) \right.$$
$$\left. + \frac{1}{\varepsilon} \left[C_F \frac{\ln z}{\bar{z}} - \frac{C_A}{2} \frac{\bar{z} + \ln z}{1-z} \right] \right)$$

We checked that above expressions are consistent with the evolution kernels of the pion LCDA [28, 29] and SCET$_I$ operator J^{B1} [13, 24]. But note, that we haven't included the diagrams with the wave function renormalizations hence the poles (4.30) and (4.31) are not exactly convolutions of the corresponding evolution kernels with the leading order coefficient functions.

Calculation of the hard integrals $(J_X)_{hard}$ can be done with standard technique and the results for each diagram are listed in the appendix. The arising IR−poles cancel in the sum with (4.30) and (4.31) as it is expected from the factorization theorem. Resulting expressions for the coefficient functions can be written as

$$\alpha_2: \ T_2^{NLO}(u,z) = \frac{C_2}{2N_c} T_D(u,z) + \frac{C_1}{N_c} T_{ND}(u,z), \tag{4.32}$$

$$\alpha_1: \ T_1^{NLO}(u,z) = \frac{C_1}{2N_c} T_D(u,z) + \frac{C_2}{N_c} T_{ND}(u,z), \tag{4.33}$$

where the subscripts D and ND can be understood as diagonal and non-diagonal contributions. Assuming for simplicity $\mu_R = \mu_F = \mu$ we obtaine:

$$
\begin{aligned}
\frac{1}{\pi} \operatorname{Im} T_D(u,z) = {} & \frac{1}{2}\left(\frac{4-u}{\bar{u}} - \frac{2\,u^2}{(1-u-z)^2} - \frac{u}{1-u-z} \right) \\
& + \left(\frac{1}{\bar{u}} - \frac{\bar{z}\,u^2}{(1-u-z)^3} \right) \ln u \\
& + \frac{z}{u-z} \ln \bar{u} + u\left(-\frac{1}{\bar{u}} - \frac{1}{u-z} + \frac{u\,\bar{z}}{(1-u-z)^3} \right) \ln \bar{z},
\end{aligned}
\tag{4.34}
$$

$$
\begin{aligned}
\operatorname{Re}\ T_D(u,z) = {} & \frac{3}{\bar{u}} \ln\left(\mu^2/m_b^2\right) + \frac{8}{\bar{u}} + \frac{1}{2}\left(-\frac{u^2+3\bar{u}}{\bar{u}^2} + \frac{2u^2}{(1-u-z)^2} + \frac{u}{1-u-z} - \frac{u}{\bar{u}^2\,\bar{z}} \right) \ln u \\
& + \left(\frac{1}{2\,\bar{u}^2\,\bar{z}} - \frac{u}{\bar{u}\,(1-u-z)^2} - \frac{u}{2\,\bar{u}^2\,(1-u-z)} \right) (1-\bar{u}\,z)\ln(1-\bar{u}\,z) \\
& + \frac{(1-u\,z)}{u\,\bar{u}\,z\,\bar{z}} \ln(1-u\,z) - \left(\frac{3}{2\,\bar{u}} + \frac{1}{u\,\bar{z}} \right) \ln\bar{u} - \frac{(2-3\,z)}{2\,\bar{u}\,\bar{z}} \ln z \\
& - \frac{\bar{z}\,(\bar{z}-3\,u)}{2\,(1-u-z)^2} \ln\bar{z} - \frac{\ln^2\bar{u}}{2\,\bar{u}} + \frac{\ln^2 u}{2\,\bar{u}} + \frac{\ln z}{\bar{u}}(\ln\bar{u} - \ln u) \\
& + \frac{\operatorname{Li}_2(\bar{u}) - \operatorname{Li}_2(u)}{\bar{u}} - \frac{\bar{z}\,u^2}{(1-u-z)^2} I(u,\bar{z}) + \frac{u\,\bar{z}}{\bar{u}} I(\bar{u},\bar{z}),
\end{aligned}
\tag{4.35}
$$

$$
\begin{aligned}
\frac{1}{\pi} \operatorname{Im} T_{ND}(u,z) = {} & \frac{C_A}{2}\left(\frac{u}{1-u-z} - \frac{1+2\,u}{2\,\bar{u}} - \frac{z}{u-z} \ln\bar{u} + \left(\frac{u^2}{(1-u-z)^2} - 1 \right) \ln u \right. \\
& \left. + \frac{z}{\bar{u}\,\bar{z}} \ln z + \left(\frac{1}{\bar{u}} - \frac{u^2}{(1-u-z)^2} + \frac{u}{u-z} \right) \ln\bar{z} \right) \\
& + C_F\left(\frac{1}{2}\left(\frac{5-u}{\bar{u}} - \frac{2\,u^2}{(1-u-z)^2} - \frac{u}{1-u-z} \right) \right. \\
& + \left(\frac{1}{\bar{u}} - \frac{u^2\,\bar{z}}{(1-u-z)^3} \right) \ln u + \frac{u\,\bar{z}}{\bar{u}\,(u-z)} \ln\bar{u} - \frac{z}{\bar{u}\,\bar{z}} \ln z \\
& \left. + \left(\frac{u^2\,\bar{z}}{(1-u-z)^3} - \frac{u}{\bar{u}} - \frac{u}{u-z} \right) \ln\bar{z} \right)
\end{aligned}
\tag{4.36}
$$

$$\text{Re } T_{ND}(u,z) = C_F \, T_{CF}(u,z) + \frac{C_A}{2} \, T_{CA}(u,z), \tag{4.37}$$

$$T_{CF}(u,z) = \frac{1}{4\bar{u}} \ln^2\left(\mu^2/m_b^2\right) + \frac{1}{\bar{u}} \left(\frac{21}{4} + \ln\bar{u} - \frac{\ln z}{\bar{z}}\right) \ln\left(\mu^2/m_b^2\right)$$

$$+ \frac{27}{2\,\bar{u}} + \frac{\pi^2 \left(4 - 3\,u - 5\,u\,z - 4\,u^2\,\bar{z}\right)}{24\,u\,\bar{u}\,\bar{z}} - \left(\frac{1}{2\,\bar{u}} + \frac{2}{u\,\bar{z}}\right) \ln\bar{u}$$

$$+ \left(-\frac{2+u}{\bar{u}} + \frac{u^2}{(1-u-z)^2} + \frac{u}{2\,(1-u-z)} + \frac{1}{\bar{u}\,\bar{z}}\right) \ln u$$

$$- \frac{3\,(3-2\,z-u\,\bar{z})}{2\,\bar{u}\,\bar{z}} \ln z + \frac{2\,(1-u\,z)}{u\,\bar{u}\,z\,\bar{z}} \ln(1-u\,z)$$

$$+ \frac{1}{2} \left(-1 - \frac{3}{\bar{u}} + \frac{2\,u^2}{(1-u-z)^2} + \frac{u}{1-u-z} - \frac{1}{u\,z^2} + \frac{3-5\,u}{u\,\bar{u}\,z}\right) \ln\bar{z}$$

$$+ \left(\frac{1}{2\,u\,\bar{u}\,z^2} - 1 - \frac{(2-u)\,u^2}{\bar{u}\,(1-u-z)^2} - \frac{u\,(u^2+2\,\bar{u})}{2\,\bar{u}^2\,(1-u-z)}\right.$$

$$\left.+ \frac{6\,u - 3 - 4\,u^2}{2\,u\,\bar{u}^2\,z} - \frac{1}{\bar{u}\,\bar{z}}\right) \ln(1-\bar{u}\,z)$$

$$+ \frac{\ln\bar{u}}{\bar{u}} \left(-(1+\bar{u})\ln u + \frac{1+\bar{u}\,\bar{z}}{\bar{z}} \ln z - \ln\bar{z}\right)$$

$$- \frac{\ln^2\bar{u}}{\bar{u}} + \frac{\ln^2 u}{2\,\bar{u}} + \frac{(2-z)}{2\,\bar{u}\,\bar{z}} \ln^2 z - \frac{(1+\bar{u})}{\bar{u}} \ln u \ln z$$

$$+ \frac{z - 2\,\bar{u}\,\bar{z}}{\bar{u}\,\bar{z}} \text{Li}_2(u) - \frac{1}{u\,\bar{u}\,\bar{z}} \text{Li}_2(\bar{u}) - \frac{1-u\,(3-z)}{u\,\bar{u}\,\bar{z}} \text{Li}_2(z)$$

$$- \frac{(1+u\,\bar{z})}{\bar{u}\,\bar{z}} \text{Li}_2(u\,z) + \left(\frac{1}{u\,\bar{u}\,\bar{z}} - 1\right) \text{Li}_2(\bar{u}\,z) + \frac{1}{\bar{u}} \text{Li}_2(\bar{z})$$

$$- \frac{u^2\,\bar{z}}{(1-u-z)^2} I(u,\bar{z}) + \frac{u\,\bar{z}}{\bar{u}} I(\bar{u},\bar{z}), \tag{4.38}$$

$$T_{CA}(u,z) = -\frac{1}{\bar{u}} \left(3 - \frac{\ln z}{\bar{z}}\right) \ln\left(\mu^2/m_b^2\right) - \frac{9}{\bar{u}} - \frac{\pi^2\,(1 - u\,(3-z))}{6\,u\,\bar{u}\,\bar{z}} \tag{4.39}$$

$$+ \frac{\ln\bar{u}}{u\,\bar{z}} + \frac{3\,(2-z)}{2\,\bar{u}\,\bar{z}} \ln z - \left(1 - \frac{5}{2\,\bar{u}} + \frac{u}{1-u-z} + \frac{1}{\bar{u}\,\bar{z}}\right) \ln u$$

$$- \frac{1-u\,z}{u\,\bar{u}\,z\,\bar{z}} \ln(1-u\,z) + \frac{(1-\bar{u}\,z)\,(\bar{z} - u\,(1-z\,\bar{z}))}{u\,\bar{u}\,(1-u-z)\,z\,\bar{z}} \ln(1-\bar{u}\,z)$$

$$- \left(1 - \frac{3}{2\,\bar{u}} + \frac{u}{1-u-z} + \frac{1-2\,u}{u\,\bar{u}\,z}\right) \ln\bar{z} - \frac{1}{2\,\bar{u}} \left(1 + \frac{1}{z}\right) \ln^2 z$$

$$+ \frac{\ln^2\bar{u}}{2} - \frac{\ln^2 u}{2} + \frac{1}{2\,\bar{u}} \ln^2\bar{z} - \ln\bar{u} \left(\frac{1}{\bar{u}\,\bar{z}} \ln z + \ln\bar{z}\right)$$

$$+ \frac{\ln u}{\bar{u}} \left(\ln z - u \ln\bar{z}\right) + \left(1 - \frac{1+u\,\bar{z}}{\bar{u}\,\bar{z}}\right) \text{Li}_2(u) - \left(2 - \frac{1}{u\,\bar{u}\,\bar{z}}\right) \text{Li}_2(\bar{u})$$

$$+ \frac{1}{\bar{u}} \text{Li}_2(z) + \frac{(1-3u+u\,z)}{u\,\bar{u}\,\bar{z}} \text{Li}_2(z) + \frac{(1+u\,\bar{z})}{\bar{u}\,\bar{z}} \text{Li}_2(u\,z)$$

$$+ \left(1 - \frac{1}{u\,\bar{u}\,\bar{z}}\right) \text{Li}_2(\bar{u}\,z) - \frac{1}{\bar{u}} \text{Li}_2(\bar{z}) + \frac{u^2}{1-u-z} I(u,\bar{z}) - u \, I(\bar{u},\bar{z}).$$

where we introduced convenient real function $I(u, z)$ which reads

$$I(u, z) = \frac{\ln(u/z)}{u - z} \left[\frac{1}{2} \ln(uz) - \ln(\bar{z}\,\bar{u}) - \ln(u + z) \right] + \frac{\ln(\bar{u}/\bar{z})}{u - z} \ln \left[\frac{u + z}{uz} - 1 \right]$$
$$+ \frac{1}{u - z} \left[2\mathrm{Li}_2(\bar{z}) + \mathrm{Li}_2 \left(\frac{z}{u + z} \right) + \mathrm{Li}_2 \left(\frac{z^2}{z + u\bar{z}} \right) - (z \leftrightarrow u) \right]. \quad (4.40)$$

Our results are in agreement with the kernels which have been earlier presented in the paper [15] but computed using different technical approach (dimensional regularization with evanescent operators).[6] Let us also observe that the function $I(u, z)$ introduced in eq. (4.40) naturally appears in calculations of the diagrams involving massive propagator of heavy quark. The similar structure have been also introduced in [15] and denoted as $F(z, u)$. Let us stress once more, that $I(u, z)$ is a real function as one can easily see from its definition (4.40). We have found that

$$I(u, z) = \frac{1}{z - u} \mathrm{Re}[F(1 - z, 1 - u)]. \quad (4.41)$$

We did not find any transformation to prove this equivalence analytically and checked it numerically for the several arbitrary values of the arguments.

Analytical expressions (4.34)–(4.40) for the coefficient functions $T_{1,2}^{NLO}$ represent the main technical results of our paper.

5. Numerical estimates of $B \to \pi\pi$ branching fractions

In this section we perform the numerical analysis of the branching fractions including next-to-leading corrections to the hard and jet coefficient functions. The main contribution to the branchings originate, obviously, from the real parts of amplitudes $\alpha_{1,2}$. We neglect in our estimates by electroweak penguins $\alpha_{i,EW}$ but include QCD penguins $\alpha_4^{u,c}$, see eq. (2.1), in the form presented in [9].

Consider first some important details in calculation $\alpha_{1,2}$ at the NLO approximation. General formula reads:

$$\alpha_i = f_0 \int_0^1 du \; V_i(u, \mu_R, \mu_h) \varphi_\pi(u, \mu_h) +$$
$$\int_0^1 du \; \varphi_\pi(u, \mu_h) \int_0^1 dz \; T_i(u, z, \mu_R, \mu_h) \, \xi_\pi^B(z, \mu_h) , \quad (5.1)$$

$$\xi_\pi^B(z, \mu_h) = \hat{f}_B f_\pi \int_0^\infty d\omega \int_0^1 dx \; \phi_B(\omega, \mu_F) J(z, x, \omega, \mu_h, \mu_F) \, \varphi_\pi(x, \mu_F). \quad (5.2)$$

where we have shown explicitly the scale dependence. To estimate the values of these amplitudes we use the following numerical input. For the coefficient functions $C_{i=1,2}(\mu_R)$ in the effective Hamiltonian (2.3) we employ the NLO results at $\mu_R = m_b$ obtained in [17] (NDR-scheme, NLO approximation):

$$C_1^{NLO}(m_b) = 1.075, \quad C_2^{NLO}(m_b) = -0.170. \quad (5.3)$$

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[6] We are grateful to S. Jäger and M. Beneke for the correspondence which helped us to fix a mistake in the expression for one diagram

where m_b denotes b-meson pole mass $m_b = 4.8$ GeV. For two others scales we accept the following values:

$$\mu_h = m_b, \quad \mu_F = \mu_{hc} = 1.5 \text{ GeV.} \quad (5.4)$$

For simplicity, the uncertainties in the scales setting will be ignored in our analysis. Then we obtained

$$\alpha_i = f_0 \int_0^1 du \, V_i(u, m_b) \varphi_\pi(u, m_b) + \int_0^1 du \, \varphi_\pi(u, m_b) \int_0^1 dz \, T_i(u, z, m_b) \, \xi_\pi^B(z, m_b) \,, \quad (5.5)$$

To compute the form factor $\xi_\pi^{B1}(z, m_b)$ we must perform evolution from scale m_b to the scale μ_{hc}:

$$\xi_\pi^B(z, m_b) = \int_0^1 dz' U_B(z, z', m_b, \mu_{hc}) \, \xi_\pi^B(z', \mu_{hc}), \quad (5.6)$$

where the evolution operator U_B is derived from the solution of the evolution equation [13, 24]:

$$\frac{d}{d \ln \mu} \xi_\pi^B(z, \mu) = - \int_0^1 dz \, [V(z, z') - \delta(z - z')\Gamma_{cusp}(\alpha_S) \ln \frac{\mu}{m_b}]\xi_\pi^B(z', \mu), \quad (5.7)$$

The explicit expression for the evolution kernels in our notation are the same as in [24], see Eq's (46,47). Let us write the solution for U_B in the form

$$U_B = U_B^{LL}(z, z') + U_B^{NLL}(z, z') + \ldots, \quad (5.8)$$

where U_B^{LL} and U_B^{NLL} can be understood as "leading log" and "next-to-leading log" approximations. Both terms are needed to perform complete calculation at next-to-leading order. The evolution of the form factor is combination of the two effects: summation of the so-called Sudakov double logarithms associated with $cusp$-anomalous dimension Γ_{cusp} and usual light-cone logarithms associated with non-local evolution kernel $V(z, z')$ (or non-local anomalous dimension). At present $\Gamma_{cusp}(\alpha_S)$ and $V(z, z')$ are known at two- and one-loop accuracy respectively. This is enough for summation of leading logarithms in U_B^{LL} but in order to compute U_B^{NLL} one has to know Γ_{cusp} at three loop and $V(z, z')$ at two loop accuracy . Because these quantities are uncalculated, we can't perform corresponding evolution. So we just neglect by this effect in our calculation.

Form factor $\xi_\pi^B(z, \mu_{hc})$ can be computed systematically order by order using factorization approach . Corresponding jet function has been computed at the next-to-leading accuracy in [12−14]. Fixing the factorization scale at this step to be equal to $\mu_F = \mu_{hc}$ we escape the summation of the large logarithms in jet function and therefore we have to provide the LCDA's $\varphi_\pi(x, \mu_{hc})$ and $\phi_B(\omega, \mu_{hc})$ at this normalization point. Assuming

decomposition $\xi^B_\pi = (\xi^B_\pi)^{LO} + \frac{\alpha_S}{2\pi}(\xi^B_\pi)^{NLO}$ we obtain:

$$\alpha_i^{HSA} = \int_0^1 du \; \varphi_\pi(u, m_b) \int_0^1 dz \; T_i(u,z) U_B(z,z') * \left[(\xi^B_\pi)^{LO}(z') + \frac{\alpha_S}{2\pi}(\xi^B_\pi)^{NLO}(z')\right] \quad (5.9)$$

$$= \int_0^1 du \; \varphi_\pi(u) \; T_i^{LO}(u) \int_0^1 dz \; U_B^{LL}(z,z') * (\xi^B_\pi)^{LO}(z') +$$

$$\int_0^1 du \; \varphi_\pi(u) \; T_i^{LO}(u) \int_0^1 dz \; U_B^{LL}(z,z') * \left[\frac{\alpha_S}{2\pi}(\xi^B_\pi)^{NLO}(z')\right] +$$

$$\int_0^1 du \; \varphi_\pi(u) \int_0^1 dz \left[\frac{\alpha_S}{2\pi} T_i^{NLO}(u,z)\right] U_B^{LL}(z,z') * (\xi^B_\pi)^{LO}(z') + \dots$$

where dots denote the neglected logarithms associated with U_B^{NLL} and higher order terms $O(\alpha_S^2)$. For simplicity by asterisk we denoted the integration with respect to z' and skip obvious scale dependence. In the first two lines of (5.9) we used that T_i^{LO} does not depend on fraction z. Then one can perform integration over external variable z

$$\int_0^1 dz \; U_B^{LL}(z,z') = U^{LL}(z') \qquad (5.10)$$

that simplifies the evolution. In the numerical calculations we have used for $U^{LL}(z')$ simple approximation that was found in [13]. Therefore we obtained

$$\int_0^1 dz \, U_B^{LL}(z,z') * (\xi^B_\pi)^{BLO}(z') = \left(-\pi\alpha_S(\mu_{hc}) \frac{C_F}{N_c} \frac{f_B f_\pi}{K_F \; \lambda_B \; m_b}\right) \int_0^1 \frac{dz}{\bar{z}} \; \varphi_\pi(z, \mu_{hc}) U^{LL}(z), \qquad (5.11)$$

where we used standard notation

$$\lambda_B^{-1} = \int \frac{d\omega}{\omega} \phi_B(\omega, \mu_{hc}) \; . \qquad (5.12)$$

Assuming the following ansatz for the pion DA amplitude

$$\varphi_\pi(u, \mu_{hc}) = 6u(1-u) + a_2^\pi(\mu_{hc}) C_2^{3/2}(2u-1) + a_4^\pi(\mu_{hc}) C_4^{3/2}(2u-1), \qquad (5.13)$$

one obtains (useful technical details can be fond in [13]):

$$\int_0^1 \frac{dz}{\bar{z}} \; \varphi_\pi(z) U^{LL}(z) = 2.72 \; (1 + \; a_2^\pi + \; a_4^\pi). \qquad (5.14)$$

The similar calculation for the next-to-leading contribution gives

$$\int_0^1 dz \, U_B^{LL}(z,z') * \left[\frac{\alpha_S}{2\pi}(\xi^B_\pi)^{NLO}(z')\right] = \int dz' \; U^{LL}(z') \frac{\alpha_S}{2\pi}(\xi^B_\pi)^{NLO}(z', \mu_{hc})$$

$$= \alpha_S f_B f_\pi / K_F \int_0^1 dz' \, U^{LL}(z') \int_0^\infty d\omega \int_0^1 dx \; \phi_B(\omega) \left[\frac{\alpha_S}{2\pi} J_{NLO}(z', x, \omega)\right] \varphi_\pi(x)$$

$$= \left(-\pi\alpha_S \frac{C_F}{N_c} \frac{f_B f_\pi}{K_F \lambda_B m_b}\right) \frac{\alpha_S}{\pi} \left((1 + a_2^\pi + a_4^\pi) \langle L^2 \rangle - [3.93 + 8.15 a_2^\pi + 10.05 a_4^\pi] \langle L \rangle + \right.$$

$$\left. [3.0 + 10.10 a_2^\pi + 16.60 a_4^\pi] \right), \qquad (5.15)$$

where we introduced

$$\langle L \rangle = \lambda_B \int_0^\infty d\omega \; \phi_B(\omega) \ln \left[\frac{m_b \omega}{\mu_{hc}^2} \right], \quad \langle L^2 \rangle = \lambda_B \int_0^\infty d\omega \; \phi_B(\omega) \ln^2 \left[\frac{m_b \omega}{\mu_{hc}^2} \right]. \qquad (5.16)$$

The last term in eq. (5.9) with the convolution of the next-to-leading coefficient function can be represented in following form:

$$\int_0^1 du \; \varphi_\pi(u) \int_0^1 dz \; \frac{\alpha_S}{2\pi} T_i^{NLO}(u,z) \int_0^1 dz' U_B^{LL}(z,z') (\xi_\pi^B)^{LO}(z',\mu_{hc}) = \qquad (5.17)$$

$$= \left(-\pi \alpha_S(\mu_{hc}) \frac{C_F}{N_c} \frac{f_B f_\pi}{K_F \; \lambda_B \; m_b} \right) \frac{\alpha_S}{2\pi} \left[\sum_{m,n=0,2,4} t_i{}^{mn} a_\pi^m(m_b) a_\pi^n(\mu_{hc}) \right]$$

where the moments $t_i{}^{mn}$:

$$t_i^{mn} = \frac{C_i}{2N_c} t_D^{mn} + \frac{C_{i\pm1}}{N_c} t_{ND}^{mn}, \qquad (5.18)$$

$$t_{D(ND)}^{mn} = \int_0^1 du \; 6u\bar{u} \; C_m^{3/2}(u-\bar{u}) \int_0^1 dz \; T_{D(ND)}^{NLO}(u,z) \int_0^1 dz' U_B^{LL}(z,z') 6z \; C_n^{3/2}(z-\bar{z}) \qquad (5.19)$$

have been computed numerically. If the evolution is switched off our results are in agreement with the moments computed in [15], see equations (54,55). Performing the evolution and with the scale fixed as described above we obtained:

$$\begin{array}{lll}
t_D^{00} = 73.64 + 13.85i\pi & t_D^{02} = 66.80 + 16.70i\pi & t_D^{04} = 66.85 + 17.74i\pi \\
t_D^{20} = 27.94 + 21.22i\pi & t_D^{22} = 23.86 + 28.46i\pi & t_D^{24} = 24.44 + 32.10i\pi \\
t_D^{40} = -12.74 + 22.61i\pi & t_D^{42} = -9.11 + 31.37i\pi & t_D^{44} = -7.71 + 34.67i\pi
\end{array} \qquad (5.20)$$

$$\begin{array}{lll}
t_{ND}^{00} = -7.34 - 14.24i\pi & t_{ND}^{02} = 33.15 - 17.43i\pi & t_{ND}^{04} = 69.63 - 18.21i\pi \\
t_{ND}^{20} = -201.69 - 39.87i\pi & t_{ND}^{22} = -180.99 - 39.19i\pi & t_{ND}^{24} = -164.74 - 39.28i\pi \\
t_{ND}^{40} = -371.10 - 51.01i\pi & t_{ND}^{42} = -347.00 - 49.66i\pi & t_{ND}^{44} = -335.62 - 49.63i\pi
\end{array} \qquad (5.21)$$

In order to compute branching fractions we used the set of input parameters given in the table below. The values for the coefficient functions $C_{3,4,6,8eff}$ are taken also from [17].

For the pion LCDA we use a simple model with two Gegenbauer moments $a_{2,4}^\pi$ (5.13). Our estimates of the moments based on the results obtained in [31, 32]. The evolution of these moments from initial scale μ_{hc} to scale m_b have been computed with the next-to-leading logarithmic accuracy for the leading order contribution (the first term in (5.9)). To perform this two-loop evolution we have used the analytical results derived in [33].

For the B-meson LCDA we accept a simple model with exponential behavior which is very popular in phenomenological applications

$$\phi_B(\omega) = \frac{\omega}{\lambda_B^2} \exp\left(-\omega/\lambda_B\right).$$

f_π	131 MeV	f_B	200±30 MeV	m_B	5.28GeV
$a_2^\pi(1.5\text{GeV})$	0.25±0.15	$\lambda_B(1.5\text{GeV})$	0.35±.15 GeV	m_b^{pole}	4.8GeV
$a_4^\pi(1.5\text{GeV})$	-0.05±0.15	f_0	0.28±0.05	m_c^{pole}	1.8GeV

$|V_{ub}| \times 10^3$	$|V_{cb}| \times 10^3$	γ	τ_B, ps	$C_3(m_b)$	$C_4(m_b)$	$C_6(m_b)$	$C_{8eff}(m_b)$
$3.7^{+1.3}_{-0.8}$ [30]	41.4	62°	1.53	0.012	-0.030	-0.035	-0.143

Table 1: Numerical values of the phenomenological parameters used to compute branching fractions.

Then one can easily calculate the moments introduced in eq. (5.16):

$$\langle L \rangle = \ln\left[\frac{m_b \lambda_B}{\mu_{hc}^2}\right] - \gamma_E, \quad \langle L^2 \rangle = \ln^2\left[\frac{m_b \lambda_B}{\mu_{hc}^2}\right] - \gamma_E + \frac{\pi^2}{6},$$

where $\gamma_E = 0.577\ldots$. With the given above central value of λ_B one obtains

$$\langle L \rangle = -0.87, \quad \langle L^2 \rangle = 2.4$$

For QCD running coupling α_S we use the two loop approximation with QCD scale $\Lambda_{QCD}^{(5)} = 225$MeV. Recall, that in our numerical estimates of the branching fractions we neglect EW-penguins contributions but include QCD penguins in the NLO approximation as given in [9]. As it was observed in those papers, the values of the pion branching fractions are very sensitive to the product $|V_{ub}| f_0$. Corresponding value can be estimated from semileptonic decay $B \to \pi l \nu$ assuming monotonic behavior of the form factor $f_+(q^2)$:

$$\frac{d\Gamma(B^0 \to \pi l \nu)}{dq^2} = \frac{G_F^2}{24\pi^2}|V_{ub}|^2\left|f_+(q^2)\right|^2 p_\pi^3$$
$$= \frac{G_F^2}{24\pi^2}|V_{ub}|^2\left|f_+(q^2)\right|^2 p_\pi^3$$
$$> \frac{G_F^2}{24\pi^2}|V_{ub}|^2\left|f_0(0)\right|^2 p_\pi^3. \tag{5.22}$$

Using results obtained by BABAR collaboration in [30] for the lowest bin in $q^2 < 8\text{GeV}^2$:

$$\tau_B \int_0^8 dq^2 \frac{d\Gamma(B^0 \to \pi l \nu)}{dq^2} = \Delta Br(B^0 \to \pi l \nu) = 0.21 \pm 0.13$$

one obtains

$$10^3\,|V_{ub}|\,f_0 < \sqrt{\frac{\Delta Br(B^0 \to \pi l \nu)}{\frac{\tau_B G_F^2}{24\pi^2}\int_0^8 dq^2 p_\pi^3}} \times 10^3 = 0.72^{+0.20}_{-0.27}\,(1.0),$$

where in brackets we show the product of the central values $|V_{ub}|$ and f_0 from the table 1. In order to satisfy this requirement (at least for upper bound) we accept following values for $|V_{ub}|$ and f_0 :

$$|V_{ub}| = 0.0038, \quad f_0 = 0.23, \quad \text{with} \quad 10^3 |V_{ub}| f_0 = 0.87 \quad . \tag{5.23}$$

As one can see from table 1, such choice of the $|V_{ub}|$ and f_0 corresponds to the lowest possible value of f_0 within indicated uncertainties. First, we compute two largest branching fractions as a functions of four parameters f_B, λ_B, a_2^π and a_4^π. These results for the tree level dominant branchings $Br(B \to \pi^-\pi^+)$ and $Br(B \to \pi^-\pi^0)$ and corresponding values for $\alpha_{1,2}$ are presented in figure 4. We show all solutions which describe the experimental points changing the parameters inside the intervals indicated in the table 1. As one can observe, there exist many possible solutions that demonstrate large ambiguity due to badly known mesons parameters. For instance, we reproduce the experimental values

$$10^6 Br(B \to \pi^-\pi^+) = 5.1 \text{ (exp: } 5.0 \pm 0.4), \tag{5.24}$$

$$10^6 Br(B \to \pi^-\pi^0) = 5.51 \text{ (exp: } 5.5 \pm 0.6), \tag{5.25}$$

with $f_B = 0.23$, $\lambda_B = 0.23$, $a_2^\pi = 0.3$ and $a_4^\pi = -0.07$. Corresponding amplitudes $\alpha_{1,2}$ have following numerical structure at this point:

$$\begin{aligned}
\alpha_1/f_0 &= [1.04 + 0.012i]_V + (-0.030)_{T^{LO}*J^{LO}} + (-0.020)_{T^{LO}*J^{NLO}} \\
&\quad + (-0.035 - 0.031i)_{T^{NLO}*J^{LO}} \\
&= 0.96 - 0.019i,
\end{aligned} \tag{5.26}$$

$$\begin{aligned}
\alpha_2/f_0 &= [0.035 - 0.077i]_V + (0.19)_{T^{LO}*J^{LO}} + (0.13)_{T^{LO}*J^{NLO}} \\
&\quad + (0.028 + 0.060i)_{T^{NLO}*J^{LO}} \\
&= 0.38 - 0.020i,
\end{aligned} \tag{5.27}$$

where for convenience we presented the answers normalized to $1/f_0$. Let us briefly comment these results. The real part of the α_1 is clearly dominated by the vertex term, the corrections from the hard spectator scattering are about 5% in absolute size. As one can see from (5.26) the radiative corrections (indicated as $_{T^{LO}*J^{NLO}}$ and $_{T^{NLO}*J^{LO}}$) numerically quite large with respect to LO term $_{T^{LO}*J^{LO}}$. The relatively large value of $T^{NLO} * J^{LO}$ contribution is explained by large value of the Wilson coefficient C_1 with respect to C_2.

For the amplitude α_2 the situation for the real part is different. The vertex contribution is very small due to the compensation between tree and NLO contributions [9]. Therefore the dominant term arises from the hard spectator scattering part of the amplitude. The structure of the NLO terms here is also different: bulk of the radiative correction is due to NLO jet function. The NLO hard spectator scattering is approximately four times smaller. Hence obvious conclusion is that the total (jet+hard) NLO contribution is very important for $\text{Re}[\alpha_2]$ and almost negligible for $\text{Re}[\alpha_1]$ respectively.

The important result of the our calculation is the imaginary part of both amplitudes $\alpha_{1,2}$. Its value, in comparison with the value of imaginary part from the vertex contribution V, is quite large and has opposite sign. Therefore the resulting imaginary parts
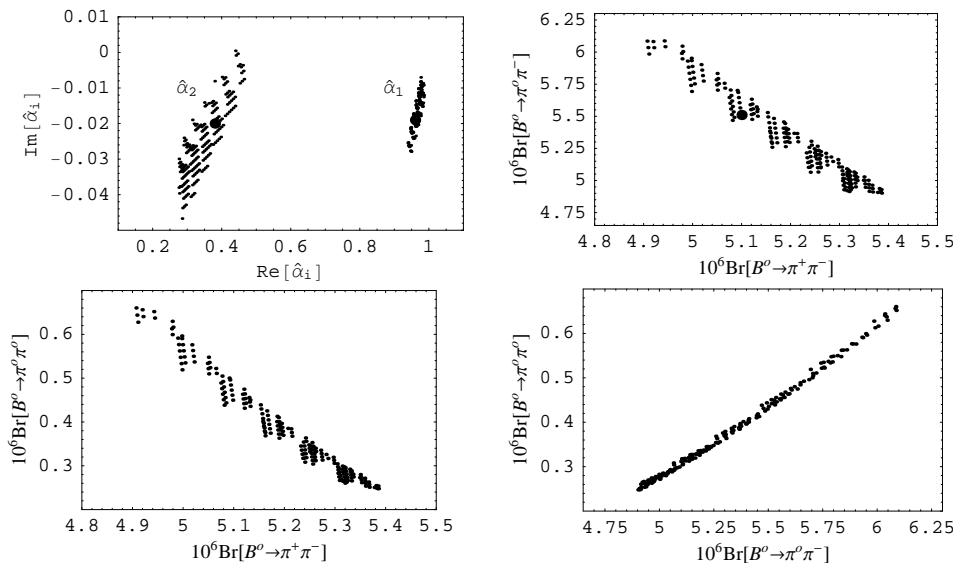
Figure 4: Branching fractions and and amplitudes $\hat{\alpha}_{1,2} = \alpha_{1,2}/f_0$ as functions of hadron input parameters $f_B(0.02)$, $\lambda_B(0.06)$, $a_2^\pi(0.06)$ and $a_4^\pi(0.06)$. The numbers in the brackets give the value of step in the numerical calculations. The large points in the upper plots correspond to the choice $f_B = 0.23$, $\lambda_B = 0.23$, $a_2^\pi = 0.30$ and $a_4^\pi = -0.07$. The values of the $\hat{\alpha}_i$ and branching fractions which lie outside of experimental interval are not shown.

are significantly modified. Such changes may produce sizable effect for the CP-violating asymmetries and therefore have to be taken into account in phenomenological analysis.

The smallness of the α_2 provides small value of the third branching $10^6 Br(B \to \pi^0\pi^0)$. Its value always remains considerably smaller than the experimental value, see two bottom plots in Fig (4). With the amplitude α_2 from eq. (5.27) we obtained

$$10^6 Br(B \to \pi^0\pi^0) = 0.45 \ (\text{exp: } 1.45 \pm 0.29)$$

that is three times smaller then the experimental value. Of course, there exist ambiguities not only due to hadronic input parameters but also in the scale setting, in quark masses and weak parameters. Such uncertainties have been already estimated in [15] and they are quite large.

On the other side it's possible to suppose that realistic explanation of the small theoretical value of the $\pi^0\pi^0$ branching can be explained by relatively large contributions of the power corrections which have been ignored in present calculations. The key observation is that $B \to \pi^0\pi^0$ amplitude has small absolute value ($\sim \alpha_S$) due to cancellation among the vertex contributions. Then it makes possible that preasymptotic effects from the power corrections are very important especially for this case. In papers [9, 15] the model for some contributions of the power corrections have been already introduced to estimate their effect. In particular, the so-called "chiral enhanced" twist-3 contributions have been considered as a source of dominant effect. In [15] such correction strongly enhances the absolute value of α_2 compare to perturbative contribution.

Similar situation, with small leading power term and large power suppressed corrections occurs in hard exclusive processes. Such scenario, as expected, is realized for the pion form factor in large Q^2 limit. In that situation leading twist perturbative contribution is small $\sim O(\alpha_S)$ and power correction may even dominate in quite large accessible range of Q^2. For the detailed discussion of this question we refer to [34] where light-cone sum rule approach have been used for analysis of the power behavior in Q^2. The other interesting observation, which was made in [34], is related to the "chiral enhanced" contributions. For the pion form factor such corrections naively could provide very strong effect as it was observed first in [35]. But in sum rule calculations it was found that such contributions turn out to be small. It might be understood, that the value of such corrections is overestimated if one uses the simple model with a cutoff of the momentum fraction to ensure convergence of the convolution integrals. If this true, then for description of $B \to \pi\pi$ decays one needs, probably, a different model of the power corrections than one used up to now. The detailed discussion of this question lies beyond the scope of present consideration and we refer to recent works dedicated to this subject [36].

6. Conclusions

We have presented the independent calculation of the next-to-leading order corrections to the graphical tree amplitudes in $B \to \pi\pi$ decays. Our analytical expressions are in agreement with results obtained in [15] using a different technical approach. The obtained results have been used for the numerical estimates of the branching fractions in BBNS approach [9]. We have found that total (hard plus jet) next-to-leading correction is relatively small for the real part of the α_1 decay amplitude and provide large contribution to the real part of α_2. In the last case the dominant effect originate from the next-to-leading contribution of the jet function. But the imaginary part which is generated by the hard coefficient function of the hard spectator scattering term is quite large and therefore can provide sizable contribution to the CP-violating asymmetries. Our estimates of the branching fractions allows to make conclusion about existence of sizable effect from power suppressed contributions, especially for branching $B \to \pi^0\pi^0$ which dominated small α_2 amplitude.

A. Structure of the divergent contributions of the Feynmann integrals

Here we briefly discuss the structure of different divergent contributions and provide results for the singular parts of the $UV-$ and $IR-$ integrals.

The $UV-$divergencies arising in the NLO diagrams are removed by the counterterms for the four-quark operators, renormalization constants of the wave functions and QCD

counterterms. Renormalization of the four fermion operators $O_{1,2}$ is given by:

$$O_1 = Z_{11}Z_\psi^{-2}O_1^{bare} + Z_{12}Z_\psi^{-2}O_2^{bare}, \tag{A.1}$$

$$O_2 = Z_{12}Z_\psi^{-2}O_1^{bare} + Z_{22}Z_\psi^{-2}O_2^{bare} \tag{A.2}$$

where 2×2 leading order matrix Z reads (see for instance [17]):

$$Z = 1 - \frac{1}{\varepsilon}\frac{\alpha_s}{4\pi}\begin{pmatrix} 2C_F - 6(C_F - C_A/2) & -3 \\ -3 & 2C_F - 6(C_F - C_A/2) \end{pmatrix} \tag{A.3}$$

and Z_ψ is the renormalization constant of the quark field in the $\overline{\text{MS}}$-scheme.

$$Z_\psi = 1 - \frac{1}{\epsilon}\frac{\alpha_s}{4\pi}C_F. \tag{A.4}$$

In addition, matrix elements have to be multiplied on the corresponding renormalization constant of the wave functions external particles. For the quark wave function such renormalization factor is defined by

$$Z_q = 1 + i\frac{d\Sigma_\psi}{d\hat{p}}\bigg|_{\hat{p}=p}, \tag{A.5}$$

where we assume off-shell $IR-$regularization and Σ_ψ denotes one-particle irreducible self-energy graphs. Such definition introduces, except $UV-$pole, the finite term:

$$\frac{1}{2}(Z_q - 1) = -\frac{1}{\epsilon}\frac{\alpha_s}{4\pi}C_F - \frac{\alpha_s}{4\pi}C_F\left[\ln\frac{\mu_R^2}{p^2} + Z_{fin}\right], \tag{A.6}$$

The pole part cancel $UV-$divergencies arising in diagrams for the matrix element. The finite term must be included in the matching. But the same definition (A.5) is used for the renormalization of the quark wave functions in the effective theory. One obtains similar contribution:

$$\frac{1}{2}(Z_q - 1)_{\text{fin}} = -\frac{\alpha_s}{4\pi}C_F\left[\ln\frac{\mu_F^2}{p^2} + Z_{fin}\right], \tag{A.7}$$

which is different from (A.6) only by renormalization scale μ_F^2. Hence, the difference of two expressions (A.6) and (A.7) which defines the contribution to the hard coefficient function from such terms is proportional to $\ln(\mu_R/\mu_F)$. We put $\mu_R = \mu_F$ that allows to avoid consideration of such terms. The same arguments can be repeated for the gluon wave functions. But situation with heavy b-quark is different. The HQET wave function of the effective filed h_v is renormalized by factor Z_h

$$Z_h = 1 + i\frac{d\Sigma_h}{d(vk)}\bigg|_{(vk)=0}, \tag{A.8}$$

where k denotes residual momentum. Then one obtains:

$$\frac{1}{2}(Z_b - 1) = \frac{\alpha_S}{4\pi}C_F\left(-\frac{1}{\varepsilon} - 2\ln\left(\frac{2(vk)}{\mu_R}\right) - 3\ln\left(\frac{\mu_R}{m_b}\right) - 2\right), \tag{A.9}$$

$$\frac{1}{2}(Z_h - 1) = \frac{\alpha_S}{4\pi}C_F\left(-\frac{1}{\varepsilon} - 2\ln\left(\frac{2(vk)}{\mu_F}\right)\right), \tag{A.10}$$

Color factors X_{col}	diagram index X
$\frac{C_i}{2N_c} + \frac{C_{i\pm1}}{N_c}\left(C_F - \frac{C_A}{2}\right)$	$A1,\ B6,\ C1,\ D6$
$\frac{C_i}{2N_c} + \frac{C_{i\pm1}}{N_c}C_F$	$A6,\ C6,\ D1$
$\frac{C_{i\pm1}}{N_c}\left(C_F - \frac{C_A}{2}\right)$	$A2, C2, D2, E6, F6, H1$
$\frac{C_{i\pm1}}{N_c}C_F$	$A4, A7, B4, C7, D4, C4, E1, E2, H6$
$\frac{C_{i\pm1}}{N_c}\frac{C_A}{2}$	$A3, C3, D3, E3, H3$

Table 2: The color factors of different graphs in figure 3

Hence the corresponding contribution to the hard coefficient function (for $\mu_R = \mu_F$)

$$(T_i)_{\Sigma_h} = \frac{\alpha_S}{2\pi}C_F\left(\frac{C_{i\pm1}}{N_c}\frac{1}{\bar{u}}\right)\left(-1 - \frac{3}{2}\ln\frac{\mu}{m_b}\right). \tag{A.11}$$

After these remarks let us provide list of the singular contributions for the integrals $(J_X)_{UV}$ defined in (4.13) and for the soft and collinear integrals (4.22), (4.24) which appear in graphs presented in figure 3. For simplicity, we shall indicate below index IR for the soft and collinear integrals and define $(X = A1, A2, \ldots)$

$$\text{pole terms}\left[(J_X)_{hard}\right] = -\text{pole terms}\left[(J_X)_{soft,coll}\right] = X_{col}JX_{IR} \tag{A.12}$$

where X_{col} denotes color factor as in (4.13). The explicit expressions for X_{col} are listed in table 2.

As usually we use notation $\bar{u} \equiv 1 - u$ and $\overline{\text{MS}}$-scheme for subtractions. For simplicity we put $\mu = m_b$. Then

$$JA6_{UV} = JC6_{UV} = JE6_{UV} = JF6_{UV} = JH1_{UV} = -JH6_{UV} = -\frac{1}{2\bar{u}\varepsilon},$$

$$JH3_{UV} = -\frac{3}{2\bar{u}\varepsilon},$$

$$JA4_{UV} = JC4_{UV} = -\frac{1}{2\varepsilon},$$

$$JB4_{UV} = JD4_{UV} = \frac{2}{\varepsilon}$$

$$, JB6_{UV} = JD6_{UV} = \frac{2}{\varepsilon\bar{u}},$$

$$JA1_{IR} = \frac{1}{\bar{u}}\left(\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(1 + i\pi - \ln[z\bar{u}]\right)\right),$$

$$JA2_{IR} = \frac{1}{\varepsilon}\frac{1}{\bar{u}}\left(-\frac{1}{2} - \frac{z\ln z}{\bar{z}}\right),$$

$$JA3_{IR} = \frac{1}{\varepsilon}\frac{1}{\bar{u}}\left(1 + \ln \bar{z}\right),$$

$$JA4_{IR} = \frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(2 + i\pi - \ln[z\bar{u}]\right),$$

$$JA7_{IR} = -\frac{1}{\varepsilon}\frac{1}{2\bar{u}},$$

$$JB4_{IR} = -\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(-2 - i\pi + \ln[zu]\right),$$

$$JB6_{IR} = \frac{1}{\bar{u}}\left(-\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\left(2 + i\pi - \ln[zu]\right)\right),$$

$$JC2_{IR} = \frac{1}{2\varepsilon^2} - \frac{1}{\varepsilon}\left(\ln u - \frac{1 - z\bar{u}}{uz}\ln\left[\frac{1 - z\bar{u}}{1 - z}\right]\right),$$

$$JC3_{IR} = \frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(-\frac{1}{2} + i\pi - \ln u - \frac{\bar{z} + 2uz}{uz}\ln \bar{z} + \frac{1 - z\bar{u}}{zu}\ln\left[1 - z\bar{u}\right]\right),$$

$$JC4_{IR} = \frac{1}{\varepsilon}\left(1 - \frac{1 - z\bar{u}}{zu}\ln\left[\frac{1 - z\bar{u}}{\bar{z}}\right]\right),$$

$$JC6_{IR} = \frac{1}{\bar{u}}\left(\frac{1}{2\varepsilon^2} + \frac{1}{\varepsilon}\left(1 - \ln u\right)\right),$$

$$JD1_{IR} = \frac{1}{\bar{u}}\left(-\frac{1}{2\varepsilon^2} + \frac{1}{\varepsilon}\ln \bar{u}\right),$$

$$JD2_{IR} = -\frac{1}{2\varepsilon^2} + \frac{1}{\varepsilon}\left(\ln \bar{u} - \frac{1 - uz}{z\bar{u}}\ln\left[\frac{1 - uz}{\bar{z}}\right]\right),$$

$$JD3_{IR} = \frac{1}{\varepsilon^2}\left(\frac{1}{2\bar{u}} - 1\right) - \frac{1}{\varepsilon}\left(i\pi - \frac{u}{2\bar{u}} - \ln \bar{u} - \frac{(1 - 2uz)}{\bar{u}z}\ln \bar{z} + \frac{(1 - uz)}{\bar{u}z}\ln[1 - uz]\right),$$

$$JD4_{IR} = \frac{1}{\varepsilon}\left(-1 + \frac{1 - uz}{\bar{u}z}\ln\frac{1 - uz}{\bar{z}}\right),$$

$$JE2 = -\frac{1}{\varepsilon}\frac{\ln \bar{u}}{u},$$

$$JE3_{IR} = \frac{1}{\bar{u}}\left(-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(-\frac{1}{2} - i\pi + \frac{\ln \bar{u}}{u} + \ln\left[u\,\bar{z}\right]\right)\right),$$

$$JE6_{IR} = \frac{1}{\varepsilon}\frac{1}{\bar{u}}\left(1 + \frac{\ln \bar{u}}{u}\right),$$

$$JF6_{IR} = \frac{1}{\bar{u}}\left(\frac{1}{2\varepsilon^2} + \frac{1}{\varepsilon}\left(1 - \ln z\right)\right),$$

$$JH1_{IR} = \frac{1}{\varepsilon}\frac{1}{\bar{u}},$$

$$JH3_{IR} = \frac{1}{\bar{u}}\left(\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\left(\frac{3}{2} + i\pi - \ln\left[\bar{z}\bar{u}\right]\right)\right). \tag{A.13}$$

Performing analysis of the main regions which contribute to the leading power accuracy we find that many $IR-$contributions cancel in certain combinations of diagrams. Important that such cancellation can be observed before computing of the integrals and therefore provides a good check for the intermediate calculations. Taking into account the color

factors of diagrams we have found following $IR-$finite combinations:

$$(JA1 + JB6 + JC6 + JD1)_{IR} = 0,$$
$$(JA4 + JB4 + JC2 + JC4 + JD2 + JD4)_{IR} = 0,$$
$$(JA1 + JB6 - JE3 + JE6 + JH1 - JH3)_{IR} = 0. \qquad (A.14)$$

The remaining $IR-$contributions can be associated with the matrix elements of the SCET operators which define LCDA φ_P^{NLO} and from factor θ^{NLO}. To obtain results for the $Z_{0,1,2}$ introduced in (A.3) we have computed following expressions:

$$\left[\frac{1}{\varepsilon}Z_1 + Z_0\right]_{coll-p'} = \frac{C_{i\pm1}}{N_c}C_F\left((JE2 + JE6 + JH1)_{coll-p'} + \int_0^1 dx'\frac{\varphi_P^{NLO}(x')}{1-x'}\right)$$
$$= \frac{C_{i\pm1}}{N_c}C_F\left(\frac{1}{\varepsilon}\frac{1}{\bar{u}}(2 + \ln\bar{u}) + \frac{\ln\bar{u}}{u}\right), \qquad (A.15)$$

$$\left[\frac{1}{\varepsilon^2}Z_2 + \frac{1}{\varepsilon}Z_1 + Z_0\right]_{s/coll-p} = \frac{C_{i\pm1}}{N_c}\Big[C_F\,(JA2 + JA7 + JF6)_{s/coll-p} + \qquad (A.16)$$
$$\frac{C_A}{2}\,(JA3 - JA2 - JC2 + JC3 - JD2 + JD3 - JF6)_{s/coll-p} + \int_0^1 dz'\theta^{NLO}(z')\Big]$$
$$= \frac{C_{i\pm1}}{N_c}\left[C_F\frac{1}{\bar{u}}\left(\frac{1}{2\varepsilon^2} - \frac{1}{\varepsilon}\frac{\ln z}{\bar{z}} - \frac{1}{2} - \frac{\ln z}{\bar{z}}\right) + \frac{C_A}{2}\frac{1}{\bar{u}}\left(\frac{1}{\varepsilon}\frac{\bar{z}+\ln z}{\bar{z}} + 1 + \frac{\ln z}{\bar{z}} + \frac{\ln\bar{z}}{z}\right)\right]$$

The pole contribution in formula (A.15) can be interpreted as convolution of the evolution kernel with the leading order coefficient function T_i^{LO}:

$$(Z_1)_{coll-p'} + \tilde{Z}_\psi = V * \tilde{t}_i^{LO} * \theta^{LO} = C_F\left(-\frac{C_{i\pm1}}{N_c}\right)\frac{1}{\bar{u}}\left(\frac{3}{2} + \ln\bar{u}\right), \qquad (A.17)$$

where in the left side of eq. (A.17) we introduced contribution from the quark field renormalization, denoted as $\tilde{Z}_\psi$. We added this term because the set diagrams in (A.15) doesn't have such contribution and therefore corresponding poles define non-trivial but not complete part of the evolution kernel V:

$$V(x,u) = C_F\left[\frac{x}{u}\theta(x < u)\left(1 + \frac{1}{u-x}\right) + \frac{1-x}{1-u}\theta(x > u)\left(1 + \frac{1}{x-u}\right)\right]_+ \qquad (A.18)$$

where plus-prescription denotes: $[F(x,u)]_+ = F(x,u) - \delta(x-u)\int_0^1 dx'F(x',u)$. The same consideration can be carried out for the poles in (A.16). Because leading order coefficient functions T_i^{LO} (3.35) are independent on the momentum fraction z', the singular (pole) part of the form factor $\theta^{NLO}(z')$ appears as integral $\int dz'\,\theta^{NLO}(z')$ and can be understood as counterterm of the local operator $J^{B1}(s = 0)$ in the effective theory. We have checked by independent calculation that expressions for the $Z_{1,2}$ in (A.16) is in agreement with the

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renormalization of the local SCET$_\text{I}$ operator $J^{B1}(s=0)$ (3.2):

$$\frac{1}{\varepsilon^2}Z_2 + \frac{1}{\varepsilon}\left[(Z_1)_{s/coll-p} + \tilde{Z}_\psi + \frac{1}{2}\tilde{Z}_A + \tilde{Z}_g\right] = \varphi_P^{LO} * T_i^{LO} * (\theta^{NLO})_{UV-pole} \tag{A.19}$$

$$= \left(-\frac{C_{i\pm 1}}{N_c}\frac{1}{\bar{u}}\right)\left(-\frac{1}{\varepsilon^2}\frac{C_F}{2}(1 + 2\varepsilon\ln[\mu/(p\bar{n})])\right.$$

$$\left.-\frac{1}{\varepsilon}\left[C_F\frac{\bar{z} - 4\ln z}{4(1-z)} + \frac{C_A}{2}\frac{\ln z}{1-z}\right]\right),$$

where we again introduced contribution from the renormalization factors for coupling $\tilde{Z}_g$ and fields $\tilde{Z}_{\psi,A}$ which necessary for complete definition of the evolution kernel. Equation (A.19) is in agreement with known results for the evolution J^{B1} obtained in [13, 24].

B. Finite contributions of the hard integrals $(J_X)_{hard}$

In the second part of appendix we present the finite contributions of the hard integrals $(J_X)_{hard}$ introduced in (4.22) and (4.23). We shall write

$$\text{finite terms}\left[(J_X)_{hard}\right] = X_{col}JX \tag{B.1}$$

As usually, we assume $\mu = m_b$ in order to simplify the formulas.

$$JA1 = \frac{2}{\bar{u}} - \frac{7\pi^2}{12\,\bar{u}} - \frac{\ln\bar{u}}{\bar{u}} + \frac{(1+z)\ln\bar{z}}{2\,\bar{u}\,z} + \frac{(3z-2)\ln z}{2\bar{u}\,\bar{z}} + \frac{\ln^2(z\,\bar{u})}{2\,\bar{u}} + \frac{i\pi\,(1-\ln[\bar{u}\,z])}{\bar{u}}\,,$$

$$JA2 = -\frac{1}{\bar{u}} + \frac{\ln\bar{u}}{2\bar{u}} + \left(-\frac{3\,z}{2} + z\ln\bar{u}\right)\frac{\ln z}{\bar{u}\,\bar{z}} + \frac{z\ln^2 z}{2\,\bar{u}\,\bar{z}} - i\pi\left(\frac{1}{2\bar{u}} + \frac{z\ln z}{\bar{u}\bar{z}}\right),$$

$$JA3 = \frac{2}{1-u} - \frac{\ln(1-u)}{1-u} + \frac{\ln(1-z)}{2\,(1-u)\,z} - \frac{\ln(1-u)\,\ln(1-z)}{1-u}$$

$$-\frac{\ln^2(1-z)}{2\,(1-u)} + \frac{i\pi\,(1+\ln\bar{z})}{1-u}\,,$$

$$JA4 = 4 - \frac{7\pi^2}{12} - \frac{3\ln[\bar{u}z]}{2} + \frac{\ln^2[z\,\bar{u}]}{2} + i\pi\left(\frac{3}{2} - \ln(\bar{u}\,z)\right),$$

$$JA6 = -\frac{i\pi}{2\,\bar{u}} - \frac{\ln\bar{z} + z - z\ln(\bar{u}\,\bar{z})}{2\,\bar{u}\,z}\,,$$

$$JA7 = -\frac{i\pi}{2\,\bar{u}} - \frac{2-\ln\bar{u}}{2\,\bar{u}}\,,$$

$$JB4 = i\pi\,\ln[u\,z] + \frac{7\pi^2}{12} + \frac{1}{2}(3 - \ln^2[uz])\,,$$

$$JB6 = \frac{1}{\bar{u}}\left(i\pi\,\ln(u\,z) + \frac{1-\log^2[u\,z]}{2} + \frac{7\pi^2}{12}\right),$$

$$JE1 = -\frac{\ln u}{2\,\bar{u}}\,,$$

$$JE2 = \frac{1}{2u}\left(-2\,i\pi\,\ln\bar{u} - 3\ln\bar{u} + \ln^2\bar{u} + 2\ln\bar{u}\,\ln\bar{z}\right),$$

$$JE3 = -\frac{1}{\bar{u}} + \frac{7\pi^2}{12\bar{u}} - \frac{\ln^2\bar{u}}{2u\bar{u}} - \frac{\ln^2[u\bar{z}]}{2\bar{u}} + \frac{\ln\bar{z}}{2\bar{u}} + \frac{3}{2\bar{u}}\ln u \ + \ln\bar{u}\left(\frac{3-2u}{2u\bar{u}} - \frac{\ln\bar{z}}{u\bar{u}}\right) -$$
$$i\pi\frac{(u - 2\ln\bar{u} - 2u\ln[u\bar{z}])}{2u\bar{u}}\Bigg) ,$$

$$JE6 = \frac{1}{2\,\bar{u}\,u}\left(4\,u + (3-u)\ln\bar{u} - \ln^2\bar{u} - u\ln\bar{z} - 2\ln\bar{u}\ln\bar{z} + i\pi\,(u + 2\ln\bar{u})\right) ,$$

$$JF6 = \frac{\pi^2}{24\bar{u}} + \frac{2}{\bar{u}} - \frac{(2-3z)\ln z}{2\,\bar{u}\bar{z}} + \frac{1}{\bar{u}}\mathrm{S}\left(\frac{\bar{z}}{z}\right) ,$$

$$JH1 = \frac{1}{2\,\bar{u}}\left(4 + i\pi - \ln(\bar{u}\,\bar{z})\right) ,$$

$$JH3 = -\frac{7\pi^2}{12\,\bar{u}} + \frac{\ln^2(\bar{u}\,\bar{z})}{2\bar{u}} - i\pi\frac{\ln(\bar{u}\,\bar{z})}{\bar{u}} ,$$

$$JH6 = \frac{1}{2\,\bar{u}}\left(1 + i\pi - \ln(\bar{u}\,\bar{z}) \right) ,$$

$$JC1 = \mathrm{Re}JC1 + i\pi\mathrm{Im}JC1,$$

$$\mathrm{Im}JC1 = \frac{\bar{z}\,(1-3\,u-z)}{2\,(1-u-z)^2} - \frac{u^2\,\bar{z}}{(1-u-z)^3}\ln\left[\frac{u}{\bar{z}}\right] ,$$

$$\mathrm{Re}JC1 = -\frac{u^2\,\bar{z}\,\mathrm{I}(u,\bar{z})}{(1-u-z)^2} - \frac{\ln u}{2}\left(1 + \frac{u}{\bar{u}^2\,\bar{z}} - \frac{u(1+u-z)}{(1-u-z)^2}\right) -$$
$$\left(\frac{1}{2} - \frac{u^2}{(1-u-z)^2} - \frac{u}{2(1-u-z)}\right)\ln\bar{z} +$$
$$\left(\frac{1}{2\bar{u}\bar{z}} - \frac{u}{(1-u-z)^2} - \frac{u}{2\bar{u}(1-u-z)}\right)\frac{(1-\bar{u}\,z)}{\bar{u}}\ln(1-\bar{u}\,z) ,$$

$$JC2 = \frac{\pi^2}{24}\left(\frac{4}{u\bar{z}} - 3\right) - \left(\frac{1}{\bar{u}} - \frac{2-u}{2\bar{u}^2\bar{z}}\right)\ln u + \left(\frac{3z-1}{2uz}\right)\ln\bar{z} -$$
$$\left(\frac{3}{2u} + \frac{2-u}{2\bar{u}^2\bar{z}} + \frac{2u-1}{2uz\bar{u}^2}\right)\ln(1-\bar{u}z) +$$
$$\left(1 - \frac{1}{u\bar{u}\bar{z}}\right)\mathrm{Li}_2(\bar{u}) + \left(1 - \frac{1}{u\bar{z}}\right)\mathrm{Li}_2(z) - \left(1 - \frac{1}{u\bar{u}\bar{z}}\right)\mathrm{Li}_2(z\bar{u}) +$$
$$\mathrm{S}\left(\frac{\bar{u}}{u}\right) + \left(\frac{1-z\bar{u}}{u\,z}\right)\left(\mathrm{S}\left(\frac{z}{\bar{z}}\right) - \mathrm{S}\left(\frac{z\bar{u}}{1-z\bar{u}}\right)\right)$$

$$JC3 = \mathrm{Re}JC3 + i\pi\mathrm{Im}JC3,$$

$$\mathrm{Im}JC3 = \frac{3}{2} - \frac{u^2}{(1-u-z)^2} + \frac{u}{2(1-u-z)} - \frac{u^3}{(1-u-z)^3}\ln\left(\frac{u}{\bar{z}}\right) - \ln(u\,\bar{z}) ,$$

$$\mathrm{Re}JC3 = 1 - \frac{7\pi^2}{12} - \frac{u^3\,\mathrm{I}(u,\bar{z})}{(1-u-z)^2} - \left(\frac{3}{2} - \frac{u^2}{(1-u-z)^2} + \frac{u}{2(1-u-z)}\right)\ln u +$$
$$+ \left(3 - 4u - \frac{3}{z} + \frac{u^2\,(z-1+3\,u)}{(1-u-z)^2}\right)\frac{\ln\bar{z}}{2u} + \frac{1}{2}\ln^2[u\bar{z}] +$$
$$+ \left(-\frac{3\bar{u}}{2u} + \frac{3-4u}{2uz\bar{u}^2} - \frac{(2-u)u^2}{\bar{u}(1-u-z)^2} - \frac{2u(2u-1)-u^3}{2\bar{u}^2(1-u-z)}\right)\ln[1-\bar{u}z]$$
$$+ \frac{(1-\bar{u}\,z)}{u\,z}\left(S\left(\frac{z}{\bar{z}}\right) - S\left(\frac{\bar{u}\,z}{1-\bar{u}\,z}\right)\right) ,$$

$$JC4 = \frac{1}{2} + \left(\frac{2-u}{uz} - \frac{1}{2uz^2} - \frac{3-2u}{2u}\right)\ln\bar{z} - \left(\frac{3}{2uz} - \frac{1}{2uz^2\bar{u}}\right)(1-z\bar{u})\ln[1-z\bar{u}] -$$

$$\frac{(1-z\bar{u})}{uz}\left(\mathrm{S}\left(\frac{z}{\bar{z}}\right) - \mathrm{S}\left(\frac{\bar{u}\,z}{1-\bar{u}\,z}\right)\right),$$

$$JC6 = \frac{\pi^2}{24\bar{u}} + \frac{3}{2\bar{u}} - \left(\frac{1}{\bar{u}} + \frac{u}{2\bar{u}^2}\right)\ln u + \frac{1}{\bar{u}}\,\mathrm{S}\left(\frac{\bar{u}}{u}\right),$$

$$JC7 = \frac{1}{2\bar{u}} + \frac{u}{2\bar{u}^2}\,\ln u,$$

$$JD1 = \mathrm{Re}JD1 + i\pi\mathrm{Im}JD1,$$

$$\mathrm{Im}JD1 = -\frac{1}{u-z} + \frac{\bar{z}}{(u-z)^2}\ln\left(\frac{\bar{z}}{\bar{u}}\right),$$

$$\mathrm{Re}JD1 = -\frac{\pi^2}{24\bar{u}} - \frac{\bar{z}\,\mathrm{I}(\bar{u},\bar{z})}{u-z} + \left(\frac{1}{u-z} - \frac{1}{u\,\bar{z}}\right)\ln\bar{u} + \frac{\ln\bar{z}}{u-z}$$

$$-\frac{(1-u\,z)\ln(1-u\,z)}{u\,\bar{z}\,(u-z)} - \frac{1}{\bar{u}}\mathrm{S}\left(\frac{u}{\bar{u}}\right),$$

$$JD2 = \frac{\pi^2}{24\bar{u}}\left(\frac{z-5-3u\bar{z}}{\bar{z}}\right) - \frac{\ln\bar{u}}{u\bar{z}} - \frac{\ln\bar{z}}{z\bar{u}} + \frac{(1-uz)}{uz\bar{u}\bar{z}}\ln[1-uz] +$$

$$\frac{1+u\bar{z}}{z\bar{u}}\left(\mathrm{Li}_2\left(u\right) + \mathrm{Li}_2\left(z\right) - \mathrm{Li}_2\left(uz\right)\right) - \mathrm{S}\left(\frac{u}{\bar{u}}\right) - \frac{1-uz}{z\bar{u}}\left(\mathrm{S}\left(\frac{z}{\bar{z}}\right) - \mathrm{S}\left(\frac{uz}{1-uz}\right)\right),$$

$$JD3 = \mathrm{Re}JD3 + i\pi\mathrm{Im}JD3,$$

$$\mathrm{Im}JD3 = \frac{1}{2\,\bar{u}} + \frac{1}{u-z} + \left(\frac{1}{\bar{u}} + \frac{\bar{u}}{(u-z)^2} + \frac{1}{u-z}\right)\ln\bar{u} + \left(2 - \frac{1}{\bar{u}} - \frac{\bar{u}}{(u-z)^2} - \frac{1}{u-z}\right)\ln\bar{z}$$

$$\mathrm{Re}JD3 = \pi^2\left(\frac{7}{12} + \frac{1}{24\bar{u}}\right) + \left(\frac{1-u\bar{u}}{\bar{u}} - \frac{uz}{\bar{u}} + \frac{\bar{u}}{u-z}\right)\mathrm{I}(\bar{u},\bar{z}) - \left(\frac{1}{2\,\bar{u}} + \frac{1}{u-z}\right)\ln\bar{u} +$$

$$\frac{1-zu}{\bar{u}z(u-z)}\ln[1-zu] - \left(\frac{u}{2\bar{u}} + \frac{1}{(u-z)} + \frac{1}{\bar{u}\,z}\right)\ln\bar{z} - \frac{1}{2}\ln^2\left[\bar{z}\bar{u}\right] -$$

$$\frac{(\bar{z}-uz)}{\bar{u}\,z}\mathrm{S}\left(\frac{z}{\bar{z}}\right) + \frac{(1-u\,z)}{\bar{u}\,z}\mathrm{S}\left(\frac{u\,z}{1-u\,z}\right),$$

$$JD4 = \frac{5}{2} + \frac{\bar{z}}{z}\ln\bar{z} + \frac{1-uz}{z\bar{u}}\left(\mathrm{S}\left(\frac{z}{\bar{z}}\right) - \mathrm{S}\left(\frac{u\,z}{1-u\,z}\right)\right),$$

$$JD6 = \mathrm{Re}JD6 + i\pi\mathrm{Im}JD6,$$

$$\mathrm{Im}JD6 = \frac{\bar{z}}{\bar{u}\,(u-z)} + \frac{\bar{z}\,(1-u\,(\bar{u}+z))}{\bar{u}\,(u-z)^2}\ln\left(\frac{\bar{u}}{\bar{z}}\right),$$

$$\mathrm{Re}JD6 = \frac{9}{2\,\bar{u}} + \left(\frac{1}{\bar{u}} - \frac{uz}{\bar{u}} + \frac{\bar{u}}{u-z}\right)\mathrm{I}(\bar{u},\bar{z}) - \left(\frac{1}{\bar{u}} + \frac{1}{u-z}\right)\ln(\bar{u}\,\bar{z})$$

$$+\frac{(1-u\,z)}{\bar{u}\,z(u-z)}\ln(1-u\,z),$$

where for brevity we used new notation $S(x)$:

$$S(x) = \frac{1}{2}\ln^2\left(1+x\right) + \int_0^1 \frac{d\alpha}{\alpha}\ln(1+x\alpha) = \mathrm{Li}_2\left(\frac{x}{1+x}\right) + \ln^2\left(1+x\right). \tag{B.2}$$

All other functions have been defined in the text.

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